LABORATORY WORK

Discrete random variable and binomial distribution law (task 1 and task 2). Continious random variable and normal distribution law (task 3)

TASK 1

There is a random variable X. Calculate numerical characteristics M(X), D(X) and σ(X), the mode Mo and the median Me. Find a distribution function F(X) of a random variable X. Draw a distribution polygon.

x_i	5	10	15	20
p_i	0,2	0,3	0,4	0,1

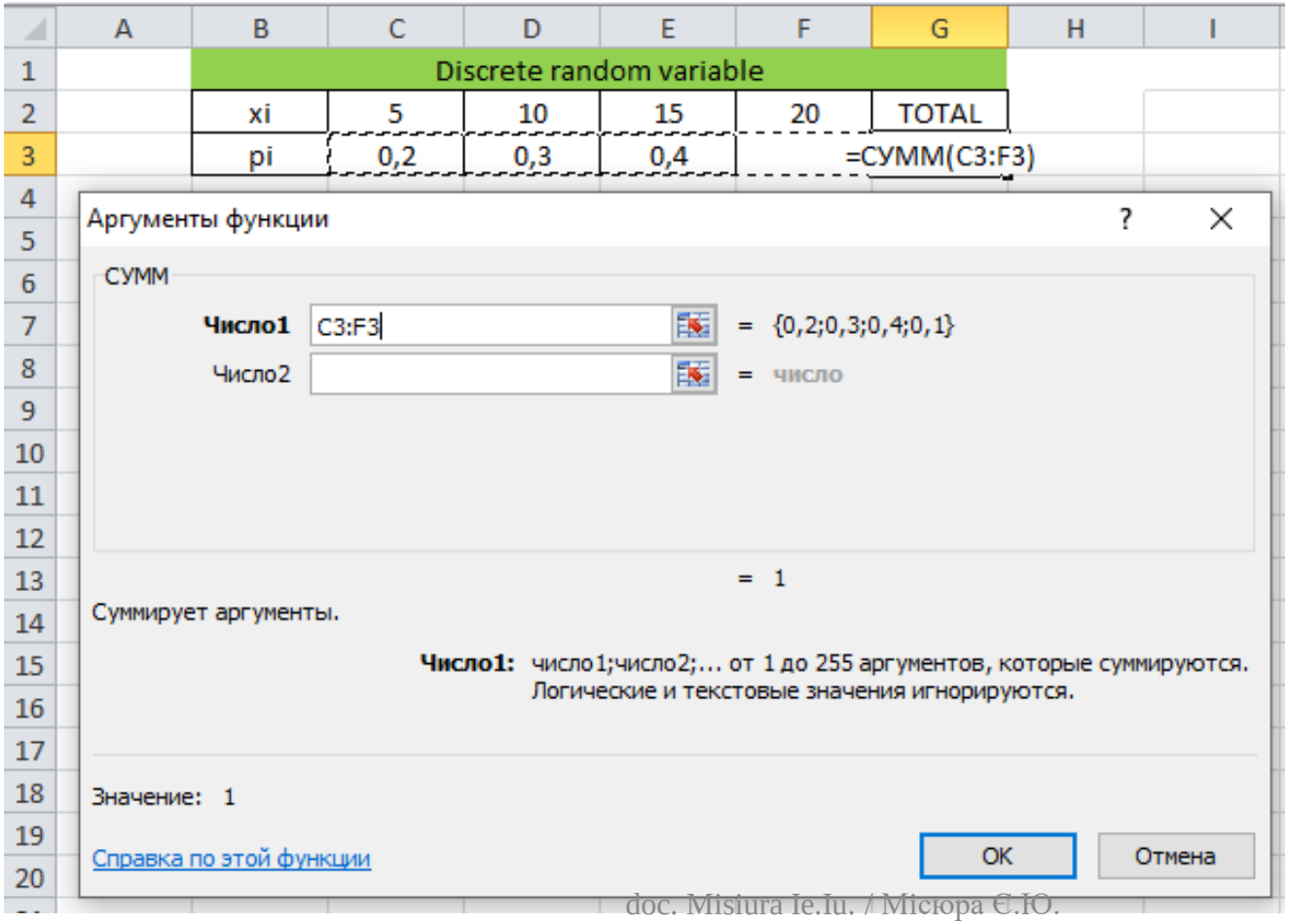
Solution in Microsoft Excel:

Fill in the initial data

	A	B	C	D	E	F	G
1		Discrete random variable					
2		xi	5	10	15	20	TOTAL
3		pi	0,2	0,3	0,4	0,1	

Solution in Microsoft Excel:

Calculate the sum of all probabilities:



Solution in Microsoft Excel:

Calculate the sum of all probabilities:

	A	B	C	D	E	F	G
1		Discrete random variable					
2		xi	5	10	15	20	TOTAL
3		pi	0,2	0,3	0,4	0,1	1

Mathematical expectation

$$M(X) = \sum_{i=1}^{n} x_i \cdot p_i = x_1 \cdot p_1 + x_2 \cdot p_2 + \ldots + x_n \cdot p_n$$

Solution in Microsoft Excel:

Calculate the products xi*pi

	A	B	C	D	E	F	G
1		Discrete random variable					
2		xi	5	10	15	20	TOTAL
3		pi	0,2	0,3	0,4	0,1	1
4		xi*pi	=C2*C3				

Solution in Microsoft Excel:

Calculate the products xi*pi

	A	B	C	D	E	F	G
1		Discrete random variable					
2		xi	5	10	15	20	TOTAL
3		pi	0,2	0,3	0,4	0,1	1
4		xi*pi	1	3	6	2	

Solution in Microsoft Excel:

Calculate the mathematical expectation M(X)

	A	B	C	D	E	F	G	H	I
1		Discrete random variable							
2		xi	5	10	15	20	TOTAL		
3		pi	0,2	0,3	0,4	0,1	1		
4		xi*pi	1	3	6		=СУММ(C4:F4)		
5									

СУММ(число1; [число2]; ...)

Solution in Microsoft Excel:

Calculate the mathematical expectation M(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)

Variance (the first formula)

$$D(X) = \sum_{i=1}^{n} \left(x_i - M(X)\right)^2 \cdot p_i$$

Solution in Microsoft Excel:

Calculate the difference xi - M(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	=C2-G4					

Solution in Microsoft Excel:

Calculate the difference xi - M(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		

Solution in Microsoft Excel:

Calculate the square of the difference xi - M(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	=C5^2					

Use keys SHIFT and 6 in order to get ^

Solution in Microsoft Excel:

Calculate the square of the difference xi - M(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		

Solution in Microsoft Excel:

Calculate the product (xi-M(X))^2*pi

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	=C6*C3					

Solution in Microsoft Excel:

Calculate the product (xi-M(X))^2*pi

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4		

Solution in Microsoft Excel:

Calculate the variance D(X)

	A	B	C	D	E	F	G	H	I
1		Discrete random variable							
2		xi	5	10	15	20	TOTAL		
3		pi	0,2	0,3	0,4	0,1	1		
4		xi*pi	1	3	6	2	12	M(X)	
5		xi - M(X)	-7	-2	3	8			
6		(xi - M(X))^2	49	4	9	64			
7		(xi - M(X))^2*pi	9,8	1,2	3,6	=СУММ(C7:F7)			
8									

СУММ(**число1**; [число2]; ...)

Solution in Microsoft Excel:

Calculate the variance D(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)

Variance (the second formula)

$$D(X) = M(X^2) - [M(X)]^2$$

where

$$M(X^2) = \sum_{i=1}^{n} x_i^2 \cdot p_i$$

Solution in Microsoft Excel:

Calculate the product xi^2*pi

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	=C2^2*C3					

Solution in Microsoft Excel:

Calculate the product xi^2*pi

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40		

Solution in Microsoft Excel:

Calculate $M(X^2)$

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$

Solution in Microsoft Excel:

Let's get values

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	=G4					
13		1st D(X)						
14		2nd D(X)						

Solution in Microsoft Excel:

Let's get values

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	=G7					
14		2nd D(X)						

Solution in Microsoft Excel:

Let's get values

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	21					
14		2nd D(X)	=G8-G4^2					

$$D(X) = M(X^2) - [M(X)]^2$$

Solution in Microsoft Excel:

Let's get values

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	21					
14		2nd D(X)	21					

Root-mean square deviation

$$\sigma(X) = \sqrt{D(X)}$$

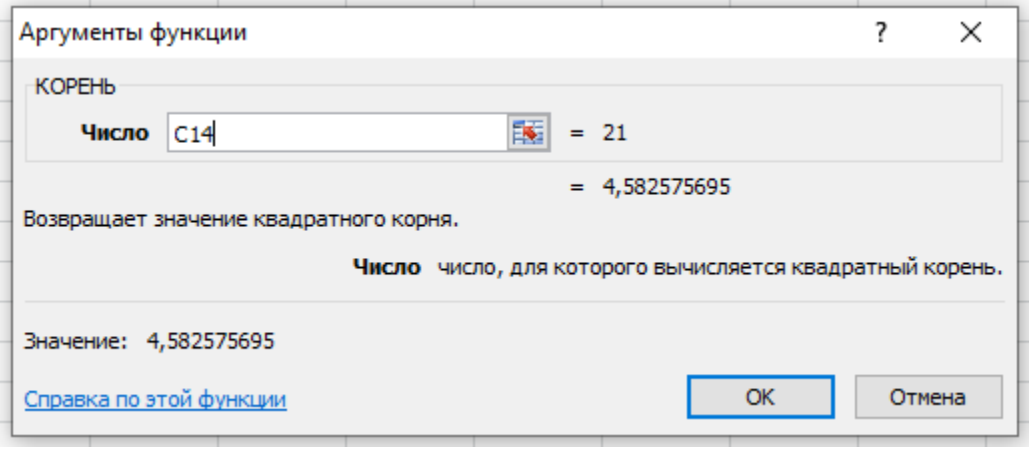
Solution in Microsoft Excel:

Calculate the root-mean square deviation σ(X)

	Discrete random variable					
xi	5	10	15	20	TOTAL	
pi	0,2	0,3	0,4	0,1	1	
xi*pi	1	3	6	2	12	M(X)
xi - M(X)	-7	-2	3	8		
(xi - M(X))^2	49	4	9	64		
(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
xi^2*pi	5	30	90	40	165	$M(X^2)$

Numerical characteristics:

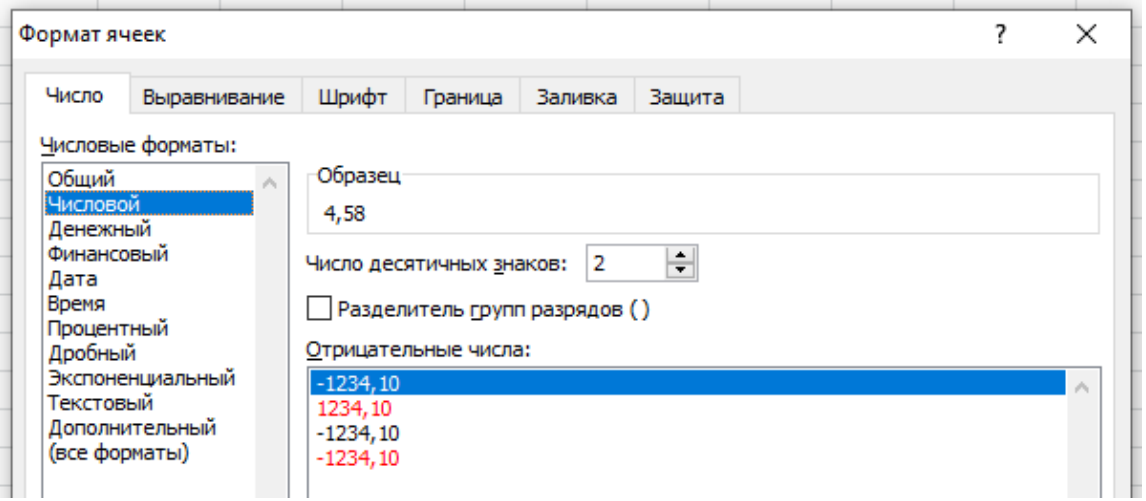
	formulas
M(X)	12
1st D(X)	21
2nd D(X)	21
σ(X)	=КОРЕНЬ(C14)



Solution in Microsoft Excel:

Calculate the root-mean square deviation σ(X)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		Discrete random variable												
2		xi	5	10	15	20	TOTAL							
3		pi	0,2	0,3	0,4	0,1	1							
4		xi*pi	1	3	6	2	12	M(X)						
5		xi - M(X)	-7	-2	3	8								
6		(xi - M(X))^2	49	4	9	64								
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)						
8		xi^2*pi	5	30	90	40	165	$M(X^2)$						
9														
10		Numerical characteristics:												
11			formulas											
12		M(X)	12											
13		1st D(X)	21											
14		2nd D(X)	21											
15		σ(X)	4,582575695											
16														
17														
18														
19														
20														
21														



Solution in Microsoft Excel:

Calculate the root-mean square deviation σ(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	21					
14		2nd D(X)	21					
15		σ(X)	4,58					

Coefficient of variation

$$\nu(X) = \frac{\sigma(X)}{M(X)} \cdot 100\,\%$$

Solution in Microsoft Excel:

Calculate the coefficient of variation v(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	21					
14		2nd D(X)	21					
15		σ(X)	4,58					
16		v(X)	=C15/C12*100					

Solution in Microsoft Excel:

Calculate the coefficient of variation v(X)

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	21					
14		2nd D(X)	21					
15		σ(X)	4,58					
16		v(X)	38,19					

A **mode** is equal to the value of x at which the probability distribution function reaches a maximum

$$M_o$$

Solution in Microsoft Excel:

Calculate the mode Mo

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	21					
14		2nd D(X)	21					
15		σ(X)	4,58					
16		v(X)	38,19					
17		Mo	=E2					

Solution in Microsoft Excel:

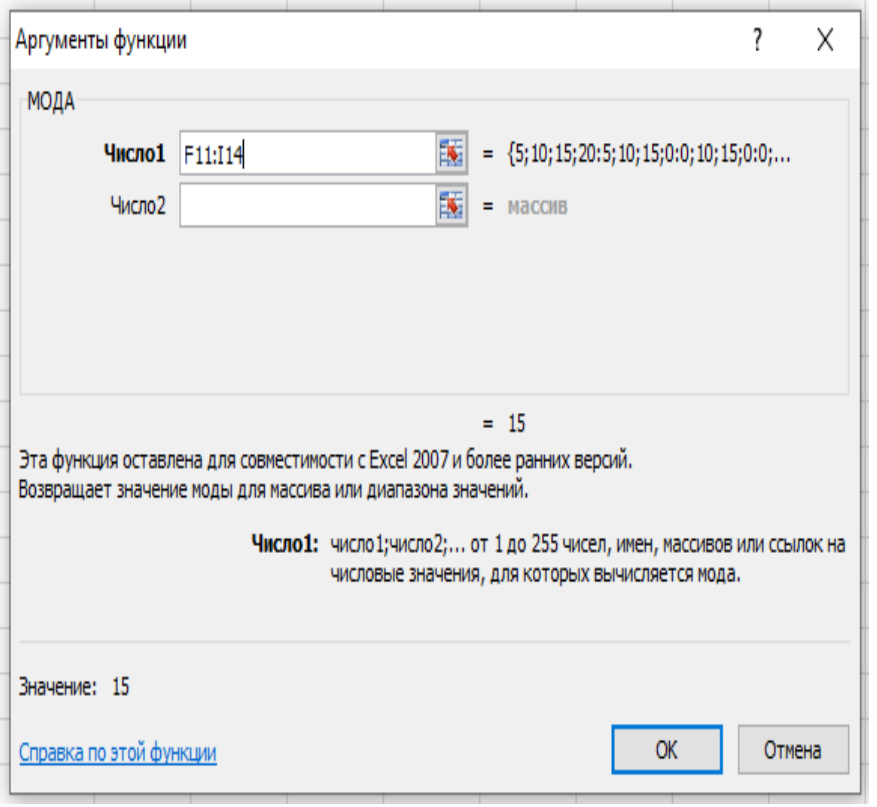
Calculate the mode Mo

	A	B	C	D	E	F	G	H
1		Discrete random variable						
2		xi	5	10	15	20	TOTAL	
3		pi	0,2	0,3	0,4	0,1	1	
4		xi*pi	1	3	6	2	12	M(X)
5		xi - M(X)	-7	-2	3	8		
6		(xi - M(X))^2	49	4	9	64		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8		xi^2*pi	5	30	90	40	165	$M(X^2)$
9								
10		Numerical characteristics:						
11			formulas					
12		M(X)	12					
13		1st D(X)	21					
14		2nd D(X)	21					
15		σ(X)	4,58					
16		v(X)	38,19					
17		Mo	15					

Solution in Microsoft Excel:

Calculate the mode Mo using the built-in function

	B	C	D	E	F	G	H
1	Discrete random variable						
2	xi	5	10	15	20	TOTAL	
3	pi	0,2	0,3	0,4	0,1	1	
4	xi*pi	1	3	6	2	12	M(X)
5	xi - M(X)	-7	-2	3	8		
6	(xi - M(X))^2	49	4	9	64		
7	(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8	xi^2*pi	5	30	90	40	165	$M(X^2)$
9							
10	Numerical characteristics:						
11		formulas			5	10	15	20
12	M(X)	12			5	10	15	
13	1st D(X)	21				10	15	
14	2nd D(X)	21					15	
15	σ(X)	4,58						
16	v(X)	38,19						
17	Mo	15		=МОДА(F11:I14)				
18								



Solution in Microsoft Excel:

Calculate the mode Mo using the built-in function

1	Discrete random variable						
2	xi	5	10	15	20	TOTAL	
3	pi	0,2	0,3	0,4	0,1	1	
4	xi*pi	1	3	6	2	12	M(X)
5	xi - M(X)	-7	-2	3	8		
6	(xi - M(X))^2	49	4	9	64		
7	(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8	xi^2*pi	5	30	90	40	165	$M(X^2)$

10	Numerical characteristics:						
11		formulas		5	10	15	20
12	M(X)	12		5	10	15	
13	1st D(X)	21			10	15	
14	2nd D(X)	21				15	
15	σ(X)	4,58					
16	v(X)	38,19					
17	Mo	15	15				

A **median** Me of a discrete random variable is the "middle" value.

$$M_e$$

Solution in Microsoft Excel:

Calculate the median Me

	A	B	C	D	E	F	G	H	I
1		Discrete random variable							
2		xi	5	10	15	20	TOTAL		
3		pi	0,2	0,3	0,4	0,1	1		
4		xi*pi	1	3	6	2	12	M(X)	
5		xi - M(X)	-7	-2	3	8			
6		(xi - M(X))^2	49	4	9	64			
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)	
8		xi^2*pi	5	30	90	40	165	$M(X^2)$	
9									
10		Numerical characteristics:							
11			formulas			5	10	15	20
12		M(X)	12			5	10	15	
13		1st D(X)	21				10	15	
14		2nd D(X)	21					15	
15		σ(X)	4,58						
16		v(X)	38,19						
17		Mo	15		15				
18		Me	=(D2+E2)/2						

Solution in Microsoft Excel:

Calculate the median Me

	A	B	C	D	E	F	G	H	I
1		Discrete random variable							
2		xi	5	10	15	20	TOTAL		
3		pi	0,2	0,3	0,4	0,1	1		
4		xi*pi	1	3	6	2	12	M(X)	
5		xi - M(X)	-7	-2	3	8			
6		(xi - M(X))^2	49	4	9	64			
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)	
8		xi^2*pi	5	30	90	40	165	$M(X^2)$	
9									
10		Numerical characteristics:							
11			formulas			5	10	15	20
12		M(X)	12			5	10	15	
13		1st D(X)	21				10	15	
14		2nd D(X)	21					15	
15		σ(X)	4,58						
16		v(X)	38,19						
17		Mo	15		15				
18		Me	12,5						

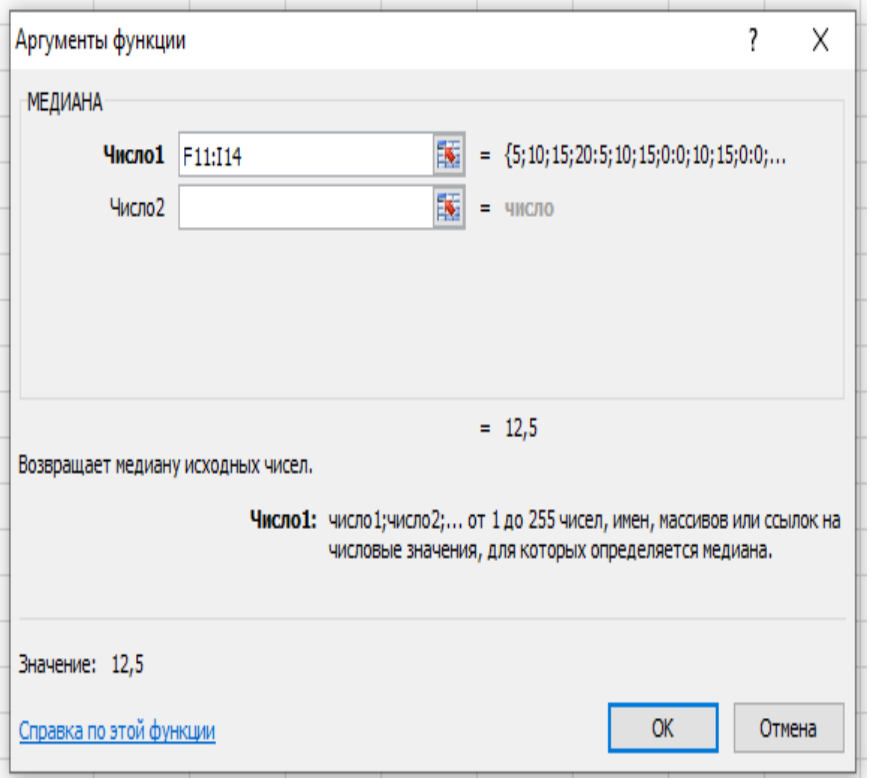
Solution in Microsoft Excel:

Calculate the median Me using the built-in function

	B	C	D	E	F	G	H
1	Discrete random variable						
2	xi	5	10	15	20	TOTAL	
3	pi	0,2	0,3	0,4	0,1	1	
4	xi*pi	1	3	6	2	12	M(X)
5	xi - M(X)	-7	-2	3	8		
6	(xi - M(X))^2	49	4	9	64		
7	(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)
8	xi^2*pi	5	30	90	40	165	$M(X^2)$

Numerical characteristics:

	formulas		5	10	15	20
M(X)	12		5	10	15	
1st D(X)	21			10	15	
2nd D(X)	21				15	
σ(X)	4,58					
v(X)	38,19					
Mo	15	15				
Me	12,5	=МЕДИАНА(F11:I14)				



Solution in Microsoft Excel:

Calculate the median Me using the built-in function

	A	B	C	D	E	F	G	H	I
1		Discrete random variable							
2		xi	5	10	15	20	TOTAL		
3		pi	0,2	0,3	0,4	0,1	1		
4		xi*pi	1	3	6	2	12	M(X)	
5		xi - M(X)	-7	-2	3	8			
6		(xi - M(X))^2	49	4	9	64			
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)	
8		xi^2*pi	5	30	90	40	165	$M(X^2)$	
9									
10		Numerical characteristics:							
11			formulas			5	10	15	20
12		M(X)	12			5	10	15	
13		1st D(X)	21				10	15	
14		2nd D(X)	21					15	
15		σ(X)	4,58						
16		v(X)	38,19						
17		Mo	15		15				
18		Me	12,5		12,5				

A distribution function of probabilities F(X)

$$F(x) = \begin{cases} 0, & x < x_1, \\ p_1, & x_1 \le x < x_2, \\ p_1 + p_2, & x_2 \le x < x_3, \\ \dots, & \\ p_1 + p_2 + \dots + p_{n-1}, & x_{n-1} \le x < x_n, \\ 1, & x \ge x_n. \end{cases}$$

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1						
4		xi*pi	1	3	6	2	12	M(X)		F(X)=			
5		xi - M(X)	-7	-2	3	8							
6		(xi - M(X))^2	49	4	9	64							
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				=C3		
4		xi*pi	1	3	6	2	12	M(X)		F(X)=			
5		xi - M(X)	-7	-2	3	8							
6		(xi - M(X))^2	49	4	9	64							
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=			
5		xi - M(X)	-7	-2	3	8							
6		(xi - M(X))^2	49	4	9	64							
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	=C3+D3		
5		xi - M(X)	-7	-2	3	8							
6		(xi - M(X))^2	49	4	9	64							
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	0,5		10 <= x < 15
5		xi - M(X)	-7	-2	3	8							
6		(xi - M(X))^2	49	4	9	64							
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	0,5		10 <= x < 15
5		xi - M(X)	-7	-2	3	8					=C3+D3+E3		
6		(xi - M(X))^2	49	4	9	64							
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	0,5		10 <= x < 15
5		xi - M(X)	-7	-2	3	8					0,9		15 <= x < 20
6		(xi - M(X))^2	49	4	9	64					=C3+D3+E3+F3		
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15	15									
18		Me	12,5	12,5									

Solution in Microsoft Excel:

Calculate values of the distribution function F (X)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	0,5		10 <= x < 15
5		xi - M(X)	-7	-2	3	8					0,9		15 <= x < 20
6		(xi - M(X))^2	49	4	9	64					1		x > = 20
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$					
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

The probability that a random variable X lies in the interval (x_1, x_2)

$$P(x_1 < X < x_2) = F(x_2) - F(x_1)$$

The probability that a random variable X lies in the interval (x_1, x_2)

$$P(x_1 < X < x_2) = P(5 < X < 13) = F(13) - F(5)$$

Solution in Microsoft Excel:

Calculate the probability P(5<x<13)

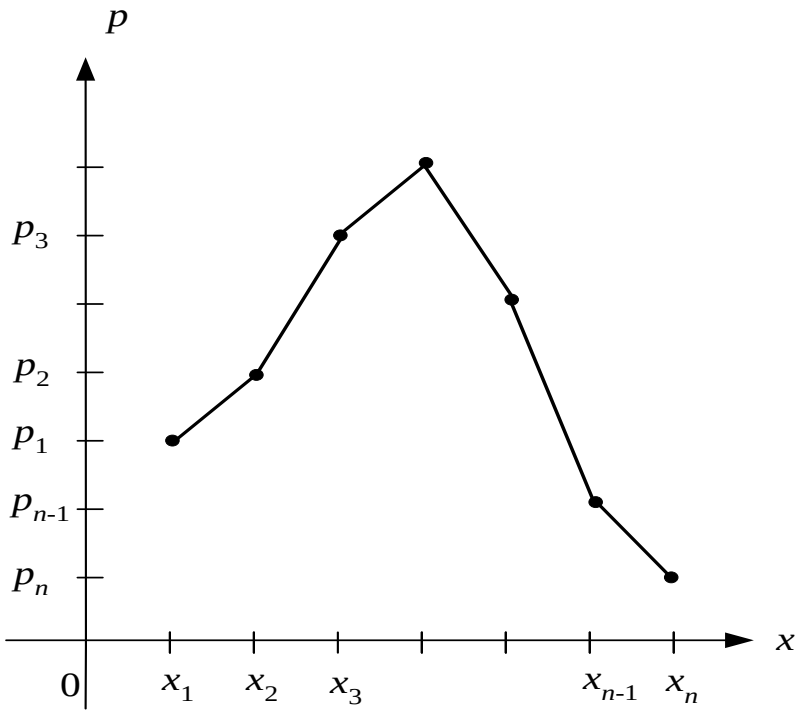
	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	0,5		10 <= x < 15
5		xi - M(X)	-7	-2	3	8					0,9		15 <= x < 20
6		(xi - M(X))^2	49	4	9	64					1		x > = 20
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$		P (5 < x < 13) =		=K4-K3	
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15	15									
18		Me	12,5	12,5									

Solution in Microsoft Excel:

Calculate the probability P(5<x<13)

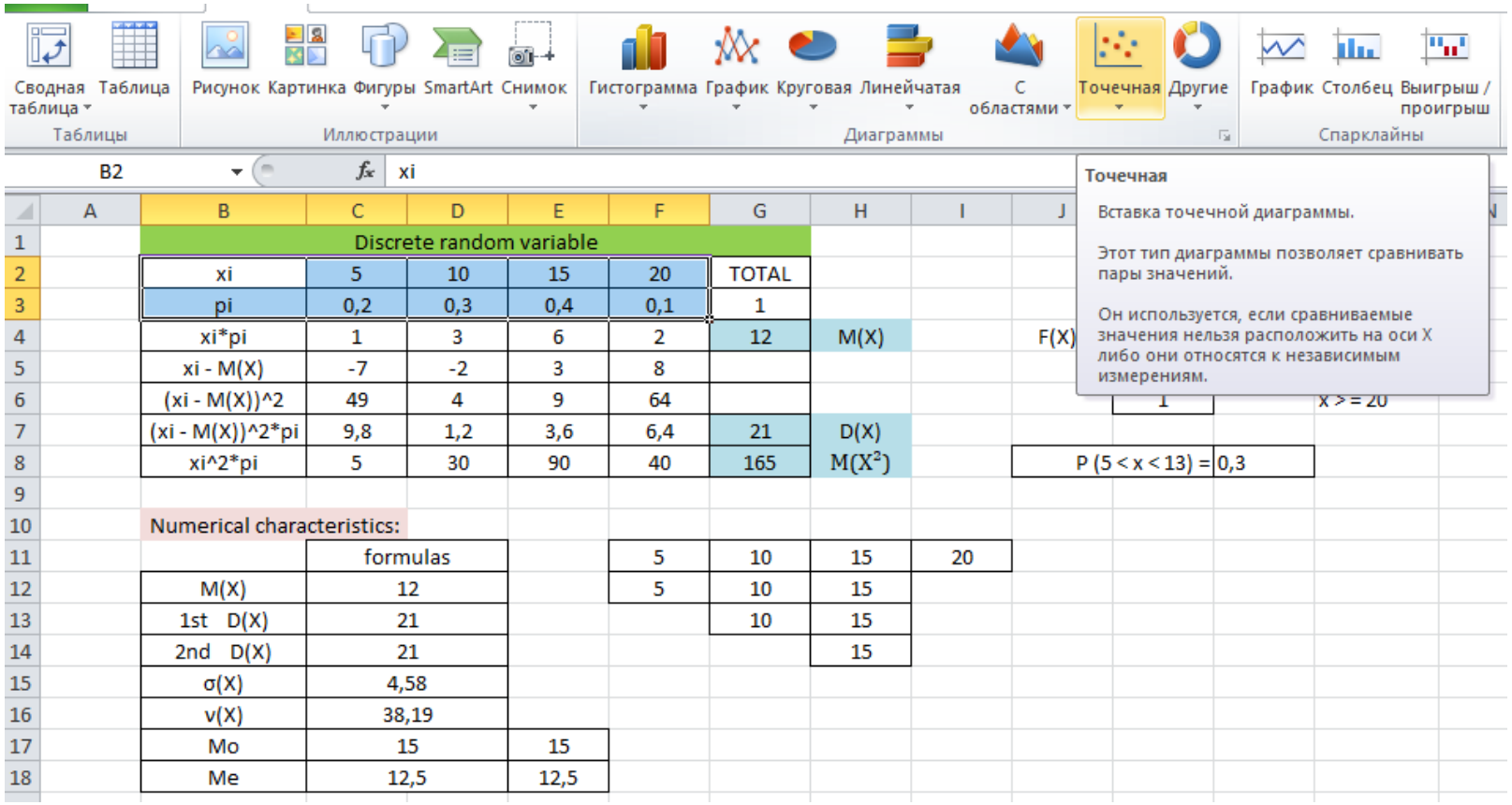
	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Discrete random variable											
2		xi	5	10	15	20	TOTAL				0		x < 5
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	0,5		10 <= x < 15
5		xi - M(X)	-7	-2	3	8					0,9		15 <= x < 20
6		(xi - M(X))^2	49	4	9	64					1		x > = 20
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)					
8		xi^2*pi	5	30	90	40	165	$M(X^2)$		P (5 < x < 13) =		0,3	
9													
10		Numerical characteristics:											
11			formulas			5	10	15	20				
12		M(X)	12			5	10	15					
13		1st D(X)	21				10	15					
14		2nd D(X)	21					15					
15		σ(X)	4,58										
16		v(X)	38,19										
17		Mo	15		15								
18		Me	12,5		12,5								

Distribution law (row) can be graphically plotted. Values of a variable are marked on ***x***-axis, the corresponding probabilities are marked on ***p***-axis. The obtained points are connected with the help of segments. It results in *a* ***distribution polygon***.



Solution in Microsoft Excel:

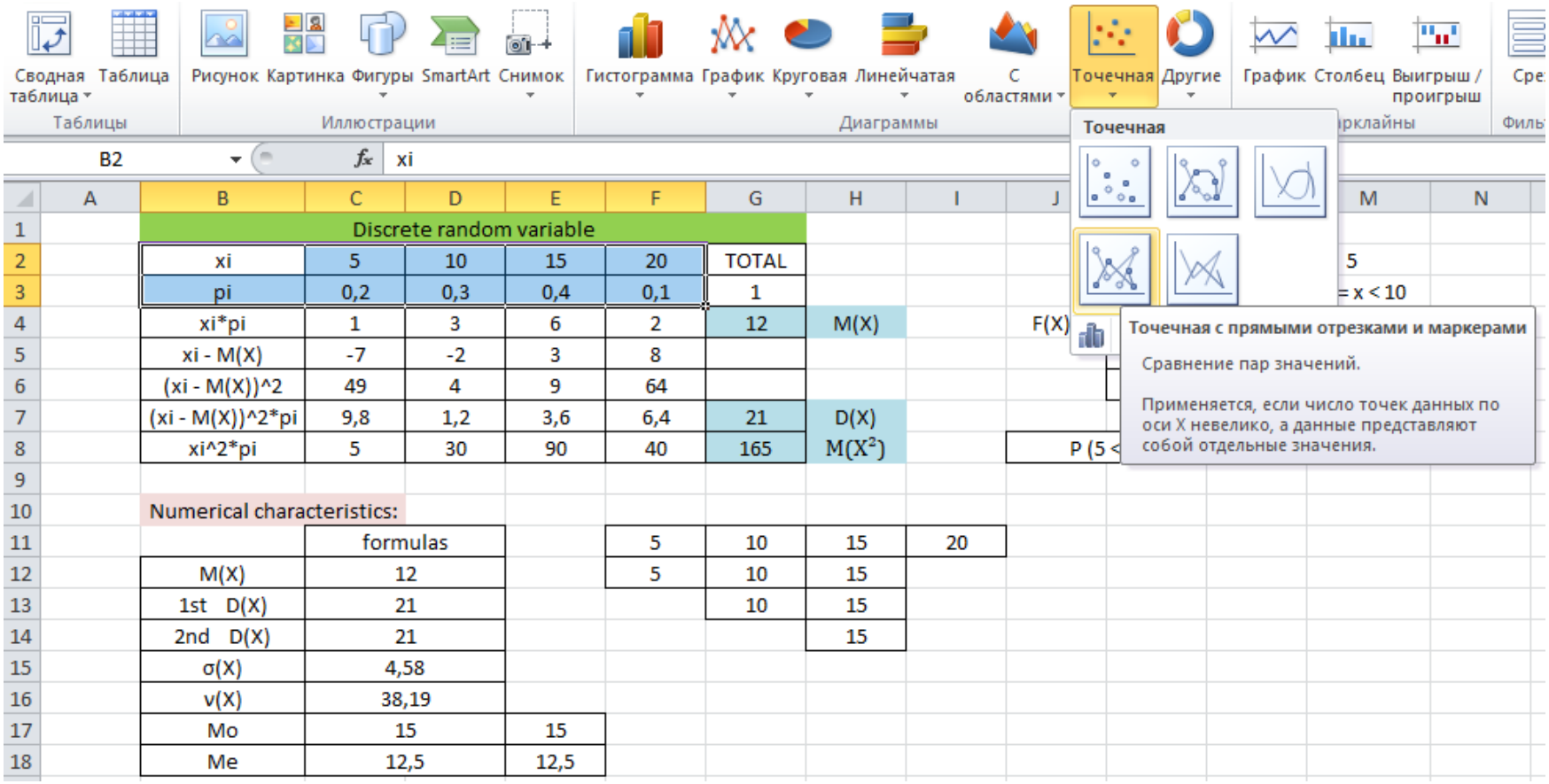
Draw the distribution polygon



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		Discrete random variable												
2		xi	5	10	15	20	TOTAL							
3		pi	0,2	0,3	0,4	0,1	1							
4		xi*pi	1	3	6	2	12	M(X)		F(X)				
5		xi - M(X)	-7	-2	3	8								
6		(xi - M(X))^2	49	4	9	64					1		X > = 20	
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)						
8		xi^2*pi	5	30	90	40	165	$M(X^2)$		P (5 < x < 13) =	0,3			
9														
10		Numerical characteristics:												
11			formulas			5	10	15	20					
12		M(X)	12			5	10	15						
13		1st D(X)	21				10	15						
14		2nd D(X)	21					15						
15		σ(X)	4,58											
16		v(X)	38,19											
17		Mo	15	15										
18		Me	12,5	12,5										

Solution in Microsoft Excel:

Draw the distribution polygon

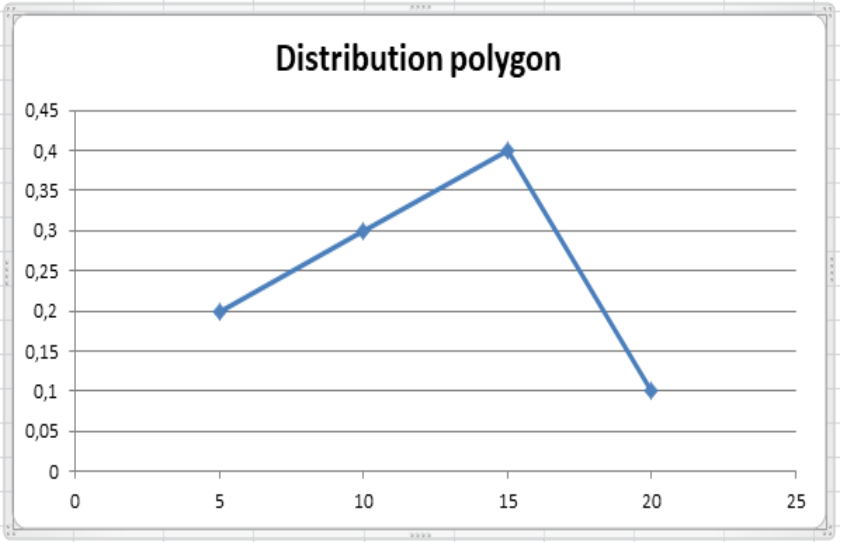


	A	B	C	D	E	F	G	H	I	J	M	N
1		Discrete random variable										
2		xi	5	10	15	20	TOTAL				5	
3		pi	0,2	0,3	0,4	0,1	1				= x < 10	
4		xi*pi	1	3	6	2	12	M(X)		F(X)		
5		xi - M(X)	-7	-2	3	8						
6		(xi - M(X))^2	49	4	9	64						
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)				
8		xi^2*pi	5	30	90	40	165	$M(X^2)$		P (5 <		
9												
10		Numerical characteristics:										
11			formulas			5	10	15	20			
12		M(X)	12			5	10	15				
13		1st D(X)	21				10	15				
14		2nd D(X)	21					15				
15		σ(X)	4,58									
16		v(X)	38,19									
17		Mo	15	15								
18		Me	12,5	12,5								

Solution in Microsoft Excel:

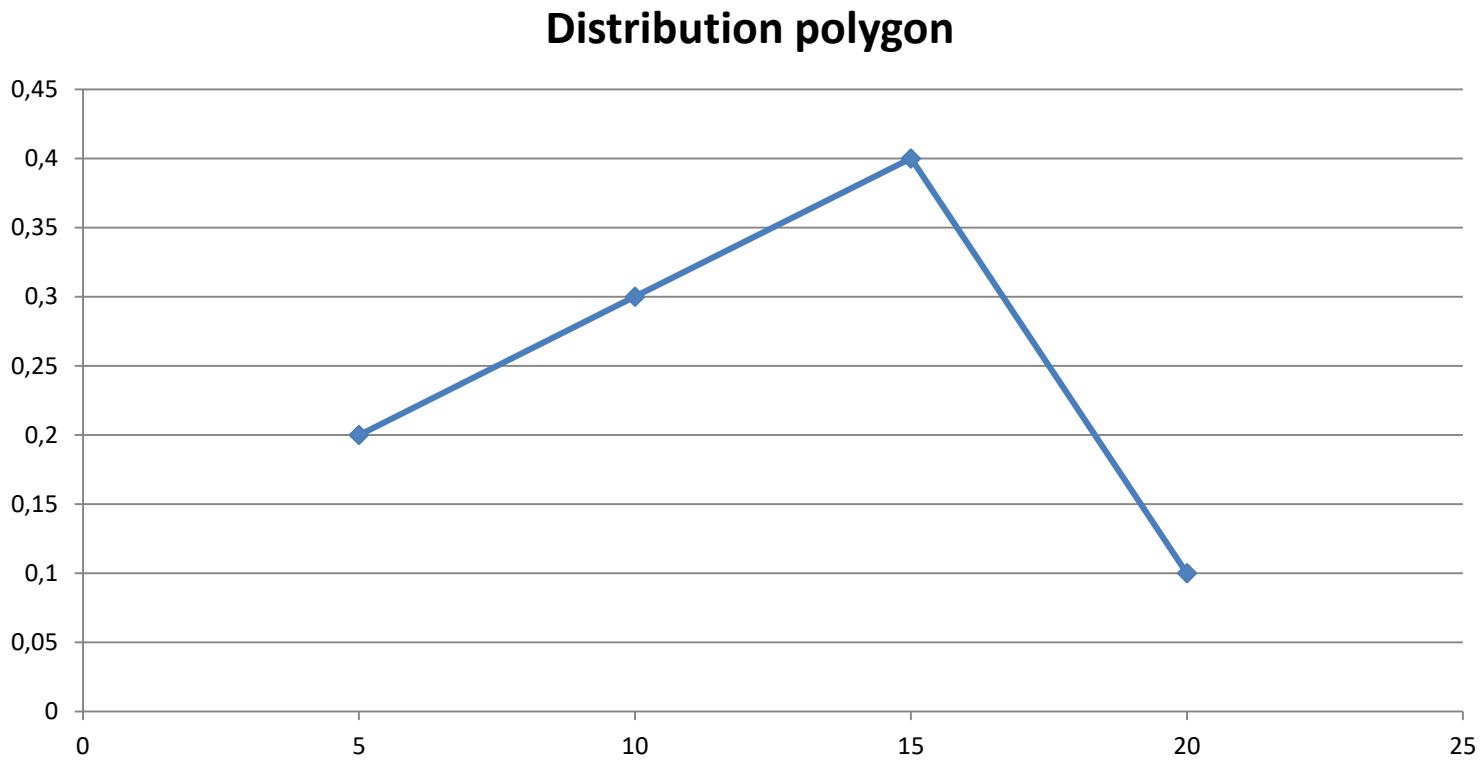
Draw the distribution polygon

1		Discrete random variable															
2		xi	5	10	15	20	TOTAL				0		x < 5				
3		pi	0,2	0,3	0,4	0,1	1				0,2		5 <= x < 10				
4		xi*pi	1	3	6	2	12	M(X)		F(X)=	0,5		10 <= x < 15				
5		xi - M(X)	-7	-2	3	8					0,9		15 <= x < 20				
6		(xi - M(X))^2	49	4	9	64					1		x > = 20				
7		(xi - M(X))^2*pi	9,8	1,2	3,6	6,4	21	D(X)									
8		xi^2*pi	5	30	90	40	165	$M(X^2)$		P (5 < x < 13) =	0,3						
9																	
10		Numerical characteristics:															
11			formulas			5	10	15	20								
12		M(X)	12			5	10	15									
13		1st D(X)	21				10	15									
14		2nd D(X)	21					15									
15		σ(X)	4,58														
16		v(X)	38,19														
17		Mo	15	15													
18		Me	12,5	12,5													



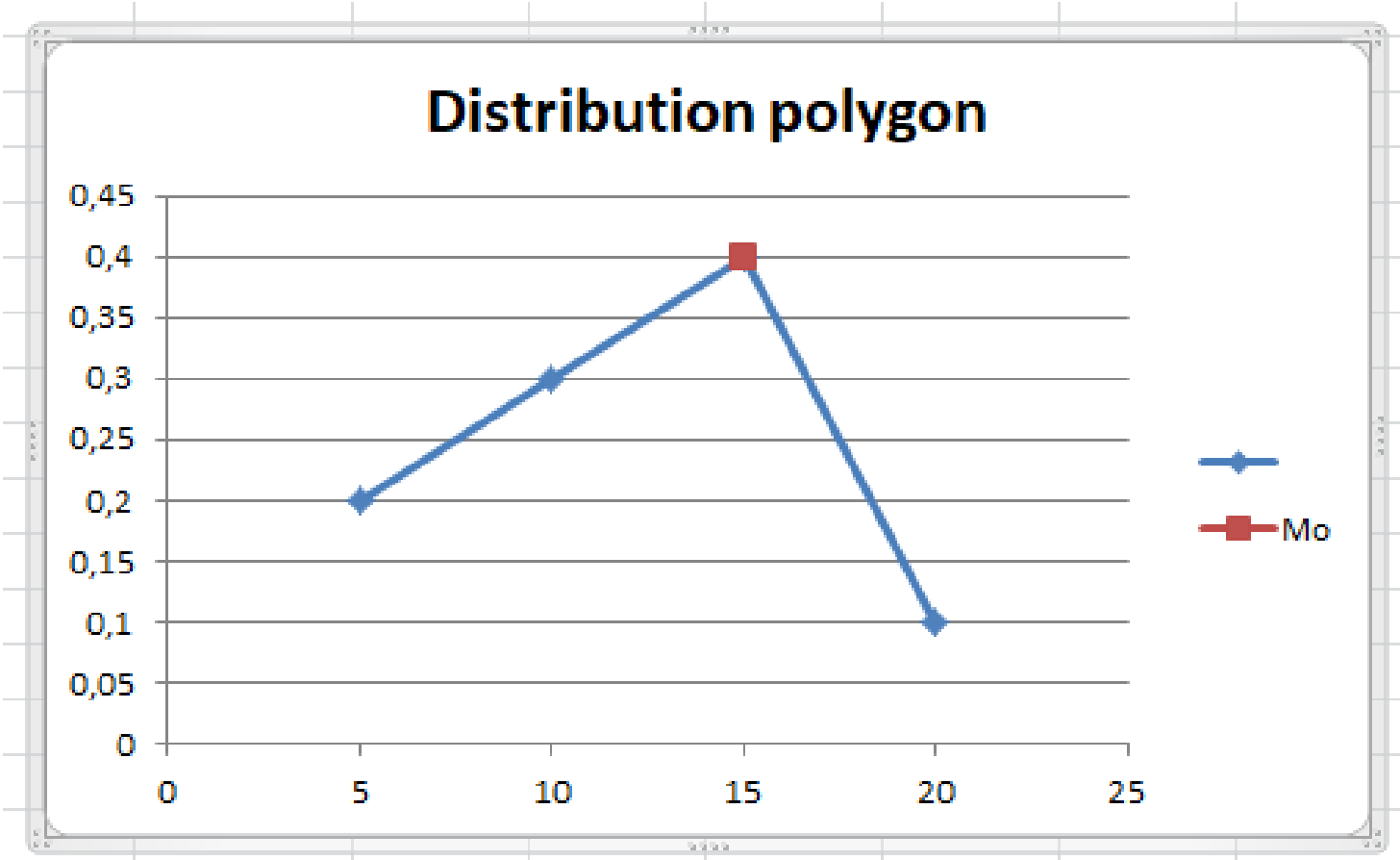
Solution in Microsoft Excel:

Draw the distribution polygon



Solution in Microsoft Excel:

Draw the distribution polygon and Mo



TASK 2

Binomial distribution law

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

Example. The probability of passing an exam excellently for each of three students (a 5-point system) equals 0.4. Construct a distribution law of a number of excellent marks which are got by the students at the exam. Find a mathematical expectation, a variance and a root-mean square deviation of a discrete random variable. Draw the distribution polygon.

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

Solution. Let a discrete random variable be a number of students with the mark “5” (a 5-point system). It has such possible values:

$x_1 = 0$

(no student passed the exam with the mark “5”);

$x_2 = 1$

(one student passed the exam with the mark “5”);

$x_3 = 2$

(two students passed the exam with the mark “5”);

$x_4 = 3$

(three students passed the exam with the mark “5”).

$$x_1 = 0$$

$$P_3(0) = C_3^0 p^0 q^{3-0} = 1 \cdot 1 \cdot q^3 = 0.6^3 = 0.216$$

$$x_2 = 1$$

$$P_3(1) = C_3^1 p^1 q^{3-1} = 3 \cdot p \cdot q^2 = 3 \cdot 0.4 \cdot 0.6^2 = 0.432$$

$$x_3 = 2$$

$$P_3(2) = C_3^2 p^2 q^{3-2} = 3 \cdot p^2 \cdot q = 3 \cdot 0.4^2 \cdot 0.6 = 0.288$$

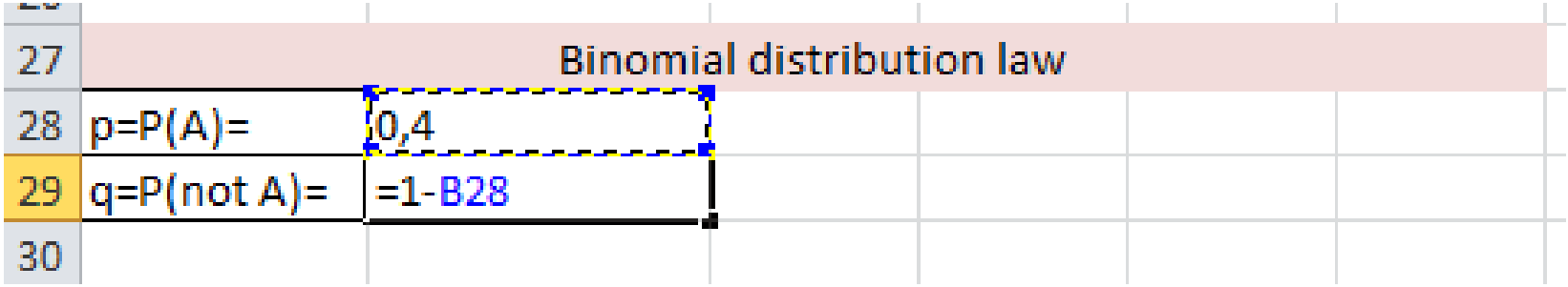
$$x_4 = 3$$

$$P_3(3) = C_3^3 p^3 q^{3-3} = 1 \cdot p^3 \cdot q^0 = 1 \cdot 0.4^3 \cdot 1 = 0.064$$

x_i	0	1	2	3
p_i	0.216	0.432	0.288	0.064

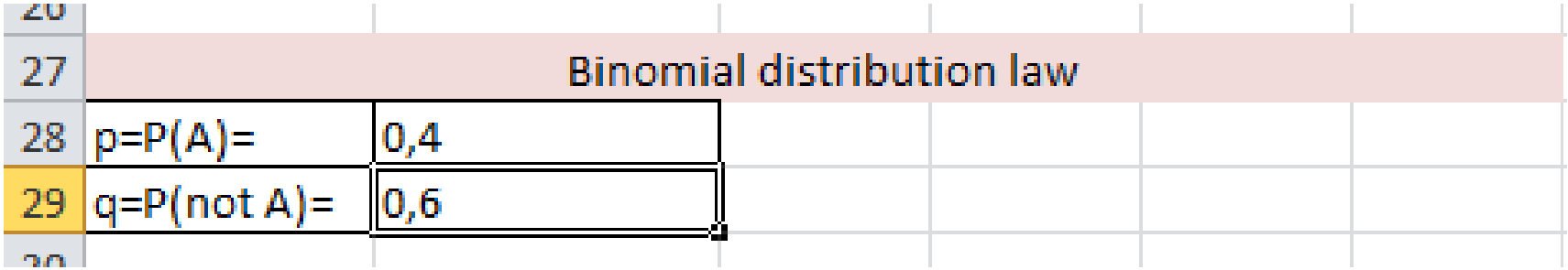
Solution in Microsoft Excel:

Define p and q



Solution in Microsoft Excel:

Define p and q



27	Binomial distribution law	
28	p=P(A)=	0,4
29	q=P(not A)=	0,6

Solution in Microsoft Excel:

Define n

27	Binomial distribution law	
28	p=P(A)=	0,4
29	q=P(not A)=	0,6
30	n=	3

Let a discrete random variable be a number of students with the mark “5” (a 5-point system). It has such possible values:

$x_1 = 0$

(no student passed the exam with the mark “5”);

$x_2 = 1$

(one student passed the exam with the mark “5”);

$x_3 = 2$

(two students passed the exam with the mark “5”);

$x_4 = 3$

(three students passed the exam with the mark “5”).

The initial data

$$n = 3 \quad p = 0.4 \quad q = 1 - p = 0.6$$

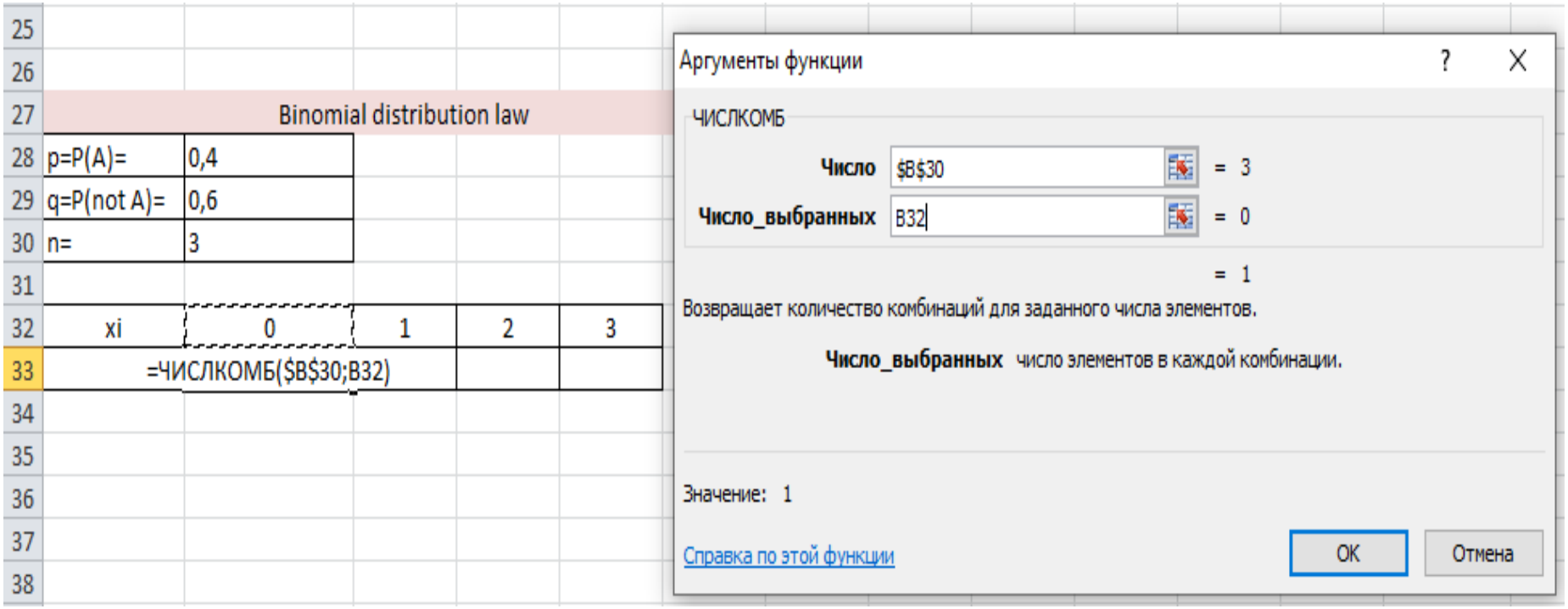
Solution in Microsoft Excel:

Define all possible values (or k)

27	Binomial distribution law				
28	p=P(A)=	0,4			
29	q=P(not A)=	0,6			
30	n=	3			
31					
32	xi	0	1	2	3
33	pi				

Solution in Microsoft Excel:

Define all probabilities



25					
26					
27	Binomial distribution law				
28	p=P(A)=	0,4			
29	q=P(not A)=	0,6			
30	n=	3			
31					
32	xi	0	1	2	3
33	=ЧИСЛКОМБ(B30;B32)				
34					
35					
36					
37					
38					

Аргументы функции

ЧИСЛКОМБ

Число B30 = 3

Число_выбранных B32 = 0

= 1

Возвращает количество комбинаций для заданного числа элементов.

Число_выбранных число элементов в каждой комбинации.

Значение: 1

Справка по этой функции

OK Отмена

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

Solution in Microsoft Excel:

Define all probabilities

ЧИСЛКОМБ | =ЧИСЛКОМБ(B30;B32)*B28^B32*B29^(B30-B32)

	A	B	C	D	E	F	G	H
25								
26								
27	Binomial distribution law							
28	p=P(A)=	0,4						
29	q=P(not A)=	0,6						
30	n=	3						
31								
32	xi	0	1	2	3			
33	=ЧИСЛКОМБ(B30;B32)*B28^B32*B29^(							
34	B30-B32)							

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

Solution in Microsoft Excel:

Define all probabilities

B33 | fx | =ЧИСЛКОМБ(B30;B32)*B28^B32*B29^(B30-B32)

	A	B	C	D	E	F	G	H
25								
26								
27	Binomial distribution law							
28	p=P(A)=	0,4						
29	q=P(not A)=	0,6						
30	n=	3						
31								
32	xi	0	1	2	3			
33	pi	0,216						

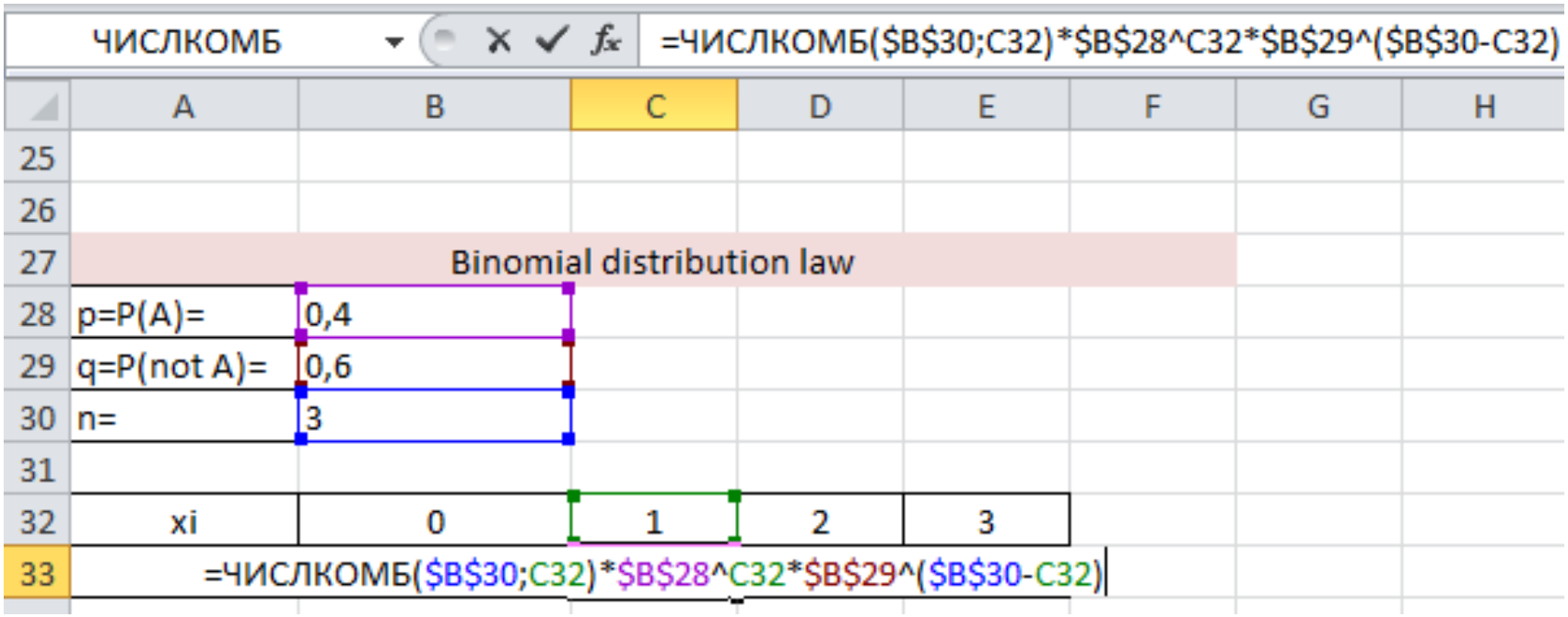
$$P_n(x=k)=C_n^k p^k q^{n-k}$$

$$x_1 = 0$$

$$P_3(0) = C_3^0 p^0 q^{3-0} = 1 \cdot 1 \cdot q^3 = 0.6^3 = 0.216$$

Solution in Microsoft Excel:

Define all probabilities



$$P_n(x=k)=C_n^k p^k q^{n-k}$$

Solution in Microsoft Excel:

Define all probabilities

27	Binomial distribution law				
28	p=P(A)=	0,4			
29	q=P(not A)=	0,6			
30	n=	3			
31					
32	xi	0	1	2	3
33	pi	0,216	0,432		

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

$$x_2 = 1$$

$$P_3(1) = C_3^1 p^1 q^{3-1} = 3 \cdot p \cdot q^2 = 3 \cdot 0.4 \cdot 0.6^2 = 0.432$$

Solution in Microsoft Excel:

Define all probabilities

27	Binomial distribution law				
28	p=P(A)=	0,4			
29	q=P(not A)=	0,6			
30	n=	3			
31					
32	xi	0	1	2	3
33	pi	=ЧИСЛКОМБ(B30;D32)*B28^D32*B29^(B30-D32)			

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

Solution in Microsoft Excel:

Define all probabilities

27	Binomial distribution law				
28	p=P(A)=	0,4			
29	q=P(not A)=	0,6			
30	n=	3			
31					
32	xi	0	1	2	3
33	pi	0,216	0,432	0,288	

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

$$x_3 = 2$$

$$P_3(2) = C_3^2 p^2 q^{3-2} = 3 \cdot p^2 \cdot q = 3 \cdot 0.4^2 \cdot 0.6 = 0.288$$

Solution in Microsoft Excel:

Define all probabilities

27	Binomial distribution law				
28	p=P(A)=	0,4			
29	q=P(not A)=	0,6			
30	n=	3			
31					
32	xi	0	1	2	3
33	pi	0			=ЧИСЛКОМБ(B30;E32)*B28^E32*B29^(B30-E32)

$$P_n(x=k)=C_n^k p^k q^{n-k}$$

Solution in Microsoft Excel:

Define all probabilities

27	Binomial distribution law				
28	p=P(A)=	0,4			
29	q=P(not A)=	0,6			
30	n=	3			
31					
32	xi	0	1	2	3
33	pi	0,216	0,432	0,288	0,064

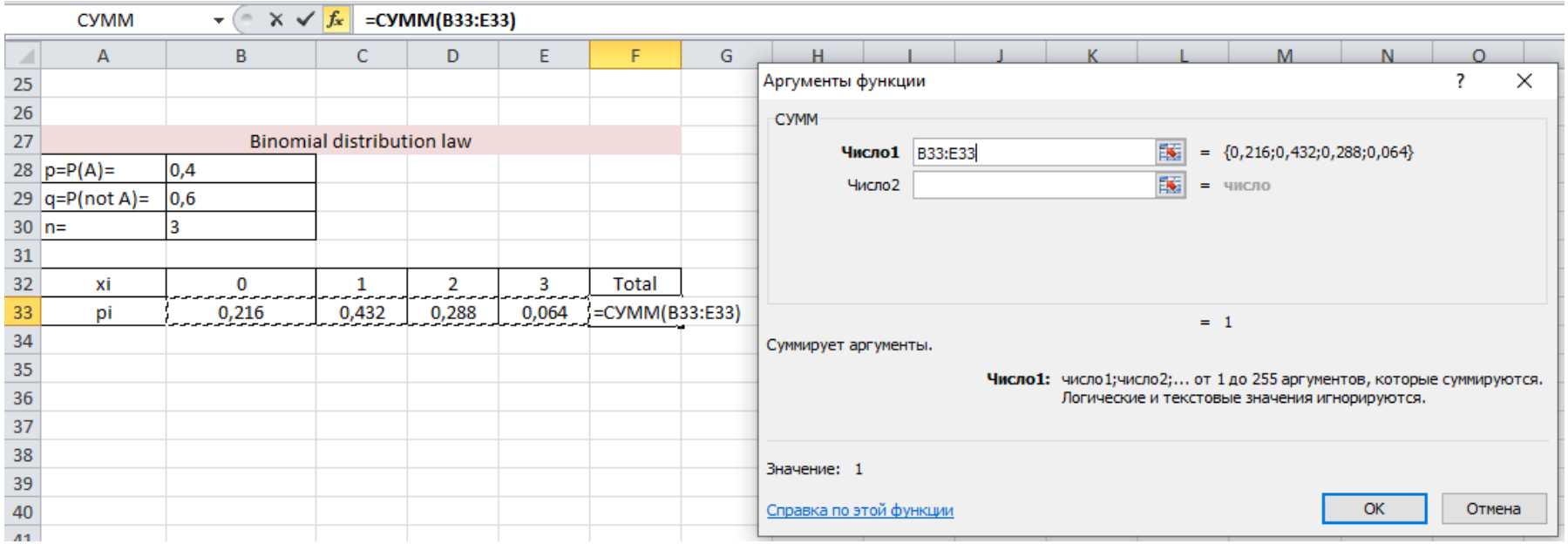
$$P_n(x=k)=C_n^k p^k q^{n-k}$$

$$x_4 = 3$$

$$P_3(3) = C_3^3 p^3 q^{3-3} = 1 \cdot p^3 \cdot q^0 = 1 \cdot 0.4^3 \cdot 1 = 0.064$$

Solution in Microsoft Excel:

Define the total



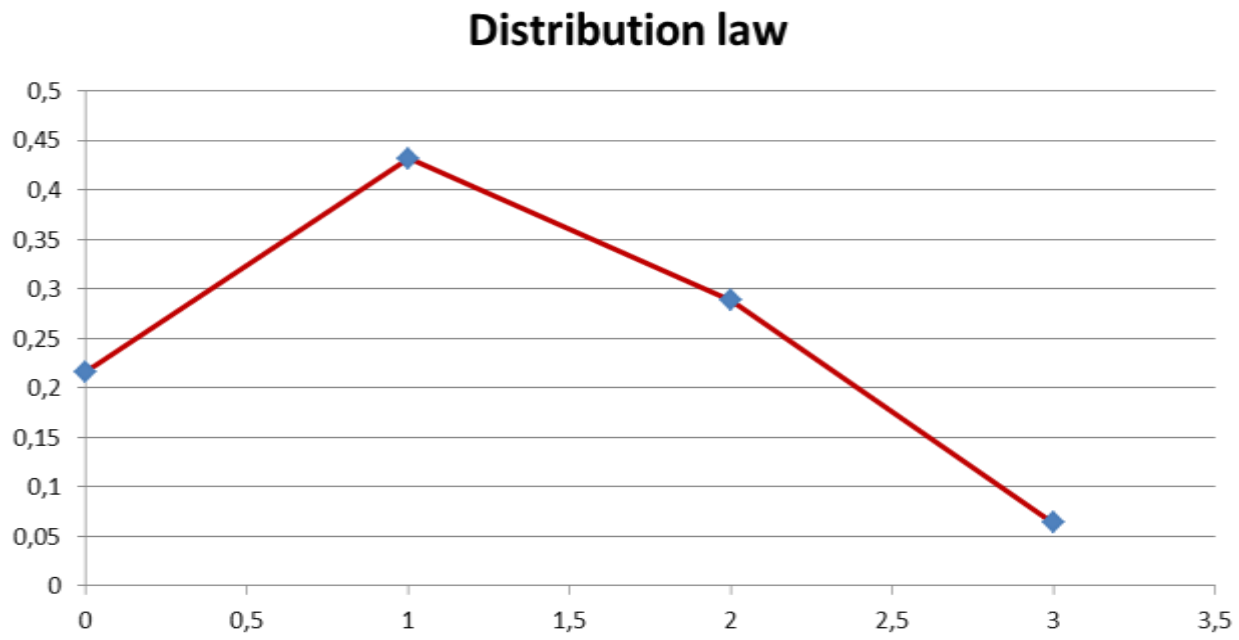
Solution in Microsoft Excel:

Define the total

27	Binomial distribution law					
28	p=P(A)=	0,4				
29	q=P(not A)=	0,6				
30	n=	3				
31						
32	xi	0	1	2	3	Total
33	pi	0,216	0,432	0,288	0,064	1

Solution in Microsoft Excel:

Draw the distribution law



Numerical characteristics

$n = 3$ $p = 0.4$ $q = 1 - p = 0.6$

$$M(X) = np = 3 \cdot 0.4 = 1.2$$

$$D(X) = npq = 3 \cdot 0.4 \cdot 0.6 = 0.72$$

$$\sigma(X) = \sqrt{npq} = \sqrt{0.72} \approx 0.85$$

Solution in Microsoft Excel:

Calculate the mathematical expectation M(X)

26	Task 2					
27		Binomial distribution law				
28	p=P(A)=	0,4				
29	q=P(not A)=	0,6				
30	n=	3				
31						
32	xi	0	1	2	3	Total
33	pi	0,216	0,432	0,288	0,064	1
34						
35	M(X)=	=B30*B28				

$$M(X) = np = 3 \cdot 0.4 = 1.2$$

Solution in Microsoft Excel:

Calculate the mathematical expectation M(X)

26	Task 2					
27	Binomial distribution law					
28	p=P(A)=	0,4				
29	q=P(not A)=	0,6				
30	n=	3				
31						
32	xi	0	1	2	3	Total
33	pi	0,216	0,432	0,288	0,064	1
34						
35	M(X)=	1,2				

$$M(X) = np = 3 \cdot 0.4 = 1.2$$

Solution in Microsoft Excel:

Calculate the variance D(X)

26	Task 2					
27	Binomial distribution law					
28	p=P(A)=	0,4				
29	q=P(not A)=	0,6				
30	n=	3				
31						
32	xi	0	1	2	3	Total
33	pi	0,216	0,432	0,288	0,064	1
34						
35	M(X)=	1,2				
36	D(X)=	=B30*B28*B29				

$$D(X) = npq = 3 \cdot 0.4 \cdot 0.6 = 0.72$$

Solution in Microsoft Excel:

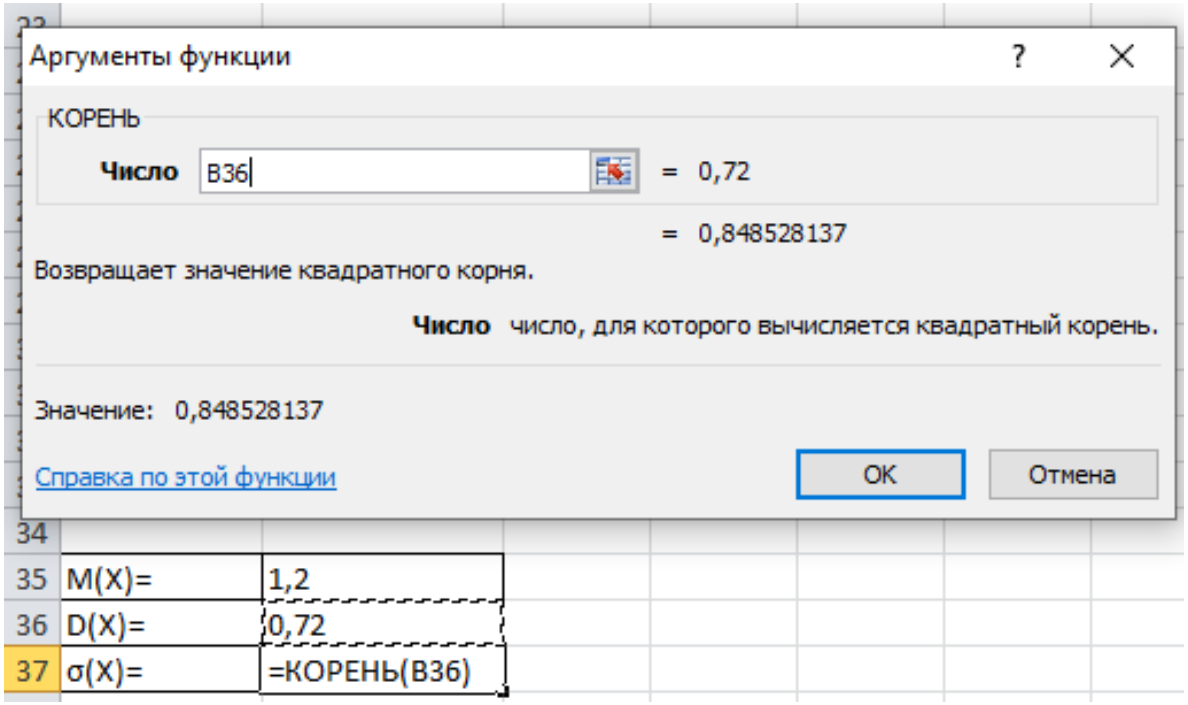
Calculate the variance D(X)

26	Task 2					
27	Binomial distribution law					
28	p=P(A)=	0,4				
29	q=P(not A)=	0,6				
30	n=	3				
31						
32	xi	0	1	2	3	Total
33	pi	0,216	0,432	0,288	0,064	1
34						
35	M(X)=	1,2				
36	D(X)=	0,72				

$$D(X) = npq = 3 \cdot 0.4 \cdot 0.6 = 0.72$$

Solution in Microsoft Excel:

Calculate the root-mean square deviation σ(X)



$$\sigma(X) = \sqrt{npq} = \sqrt{D(X)} = \sqrt{0.72} \approx 0.85$$

Solution in Microsoft Excel:

Calculate the root-mean square deviation σ(X)

26	Task 2					
27	Binomial distribution law					
28	p=P(A)=	0,4				
29	q=P(not A)=	0,6				
30	n=	3				
31						
32	xi	0	1	2	3	Total
33	pi	0,216	0,432	0,288	0,064	1
34						
35	M(X)=	1,2				
36	D(X)=	0,72				
37	σ(X)=	0,85				

$$\sigma(X) = \sqrt{npq} = \sqrt{D(X)} = \sqrt{0.72} \approx 0.85$$

LABORATORY WORK

Normal distribution law

TASK 3

Task 3. The mathematical expectation of a normal random variable equals 20, the root-mean-square deviation equals 5. a) Define the variance D(X) and the variation coefficient ν(X); b) find formulas of functions f(x) and F(x); c) plot graphs of f(x) and F(x); d) the probability that a random variable X lies in the interval (α; β), where α=13 and β=24 using two ways; e) find limits of all values of variable X on the interval with the probability 100 % using *three sigma rule* $(a \pm 3\sigma)$

SOLUTION

Solution in Microsoft Excel:

Fill in the initial data

	A	B	C	D
1	Task 1	Normal DL		
2		a	20	
3		σ	5	
4		α	13	
5		β	24	
6				

Mathematical expectation

$$M(X) = a$$

Solution in Microsoft Excel:

	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	=C2

Variance

$$D(X) = \sigma^2$$

Solution in Microsoft Excel:

	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	20
7		$D(x)=\sigma^2$	=C3^2

Root-mean square deviation

$$\sigma(X) = \sigma$$

Solution in Microsoft Excel:

Calculate the root-mean square deviation σ(X)

	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	20
7		$D(x)=\sigma^2$	25
8		σ(x)=σ	=C3

Coefficient of variation

$$\nu(X) = \frac{\sigma(X)}{M(X)} \cdot 100\,\%$$

Solution in Microsoft Excel:

Calculate the coefficient of variation v(X)

	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	20
7		$D(x)=\sigma^2$	25
8		σ(x)=σ	5
9		ν=σ(x)/Λ	=C8/C6*100

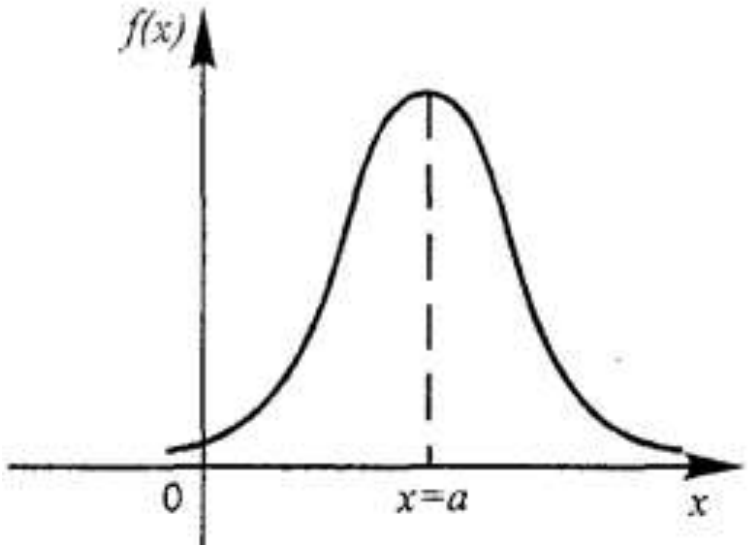
Solution in Microsoft Excel:

Calculate the coefficient of variation v(X)

	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	20
7		$D(x)=\sigma^2$	25
8		σ(x)=σ	5
9		ν=σ(x)/M(x)*100%	25

The probability density function of the normal law of distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}}$$



Let's substitute $a=20$, $\sigma^2=25$ and $\sigma=5$ and get

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-a)^2}{2\sigma^2}}$$

$$f(x)=\frac{1}{5\sqrt{2\pi}}\cdot e^{-\frac{(x-20)^2}{50}}$$

Let’s plot the graph of f(x)

$$f(x) = \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}}$$

Solution in Microsoft Excel:

Fill in values of variable X

$$f(x) = \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}}$$

	G	H	I	J	K	L	M	N	O	P
	x	-20	-10	0	10	20	30	40	50	60
	f(x)									

Solution in Microsoft Excel:

Calculate values f(x):

	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	20
7		D(x)=σ²	25
8		σ(x)=σ	5
9		v=σ(x)/M(x)*100%	25

$$f(x)=\frac{1}{5\sqrt{2\pi}}\cdot e^{-\frac{(x-20)^2}{50}}$$

$$F(x)=\int_{-\infty}^{x}\frac{1}{5\sqrt{2\pi}}\cdot e^{\frac{(x-20)^2}{50}}dx$$

	G	H	I	J	K	L	M	N	O	P
6	x	-20	-10	0	10	20	30	40	50	60
7	=1/(C8*КОРЕНЬ(2*ПИ()))*EXP(-((H6-C6)^2)/(2*C7))									
8	F(x)									

Solution in Microsoft Excel:

Calculate values f(x):

	x	-20	-10	0	10	20	30	40	50	60
=1/(C8*КОРЕНЬ(2*ПИ()))*EXP(-((H6-C6)^2)/(2*C7))										
	F(x)									

Solution in Microsoft Excel:

Calculate values f(x):

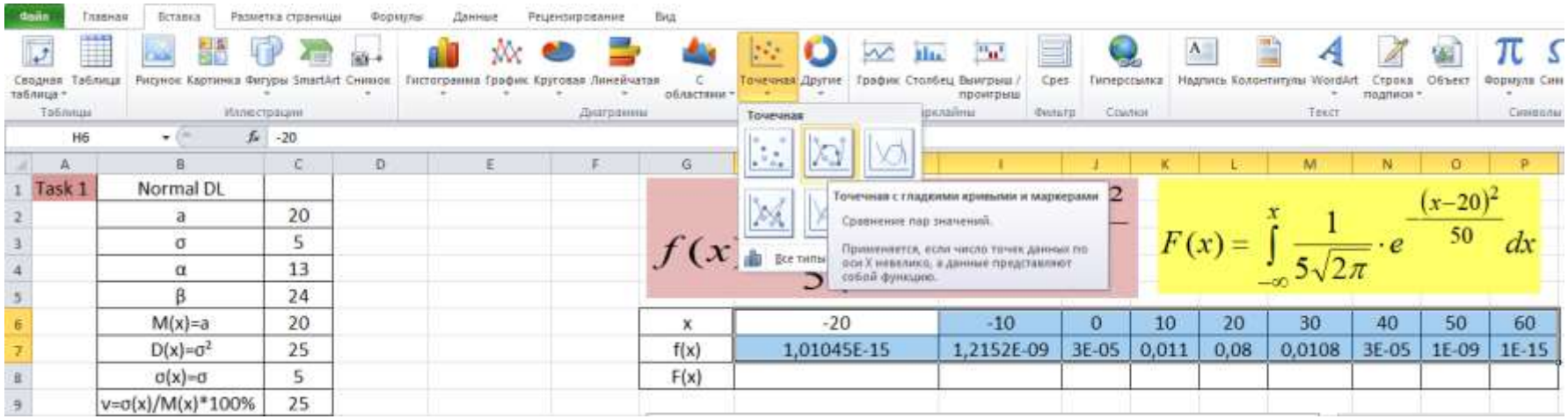
G	H	I	J	K	L	M	N	O	P

$$f(x) = \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}}$$

x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15

Solution in Microsoft Excel:

Plot the graph of f(x):

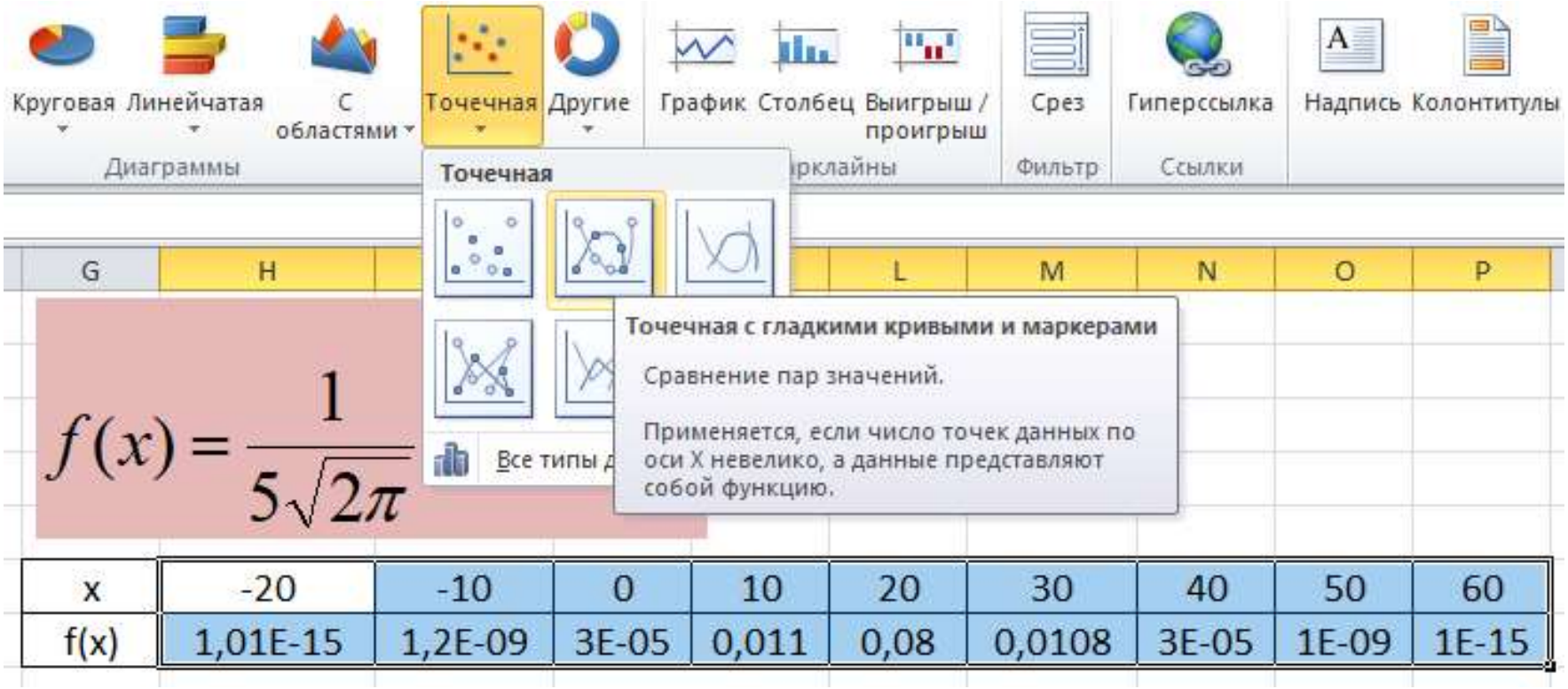


	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	20
7		D(x)=σ²	25
8		σ(x)=σ	5
9		v=σ(x)/M(x)*100%	25

x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01045E-15	1,2152E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15
F(x)									

Solution in Microsoft Excel:

Plot the graph of f(x):



$$f(x) = \frac{1}{5\sqrt{2\pi}}$$

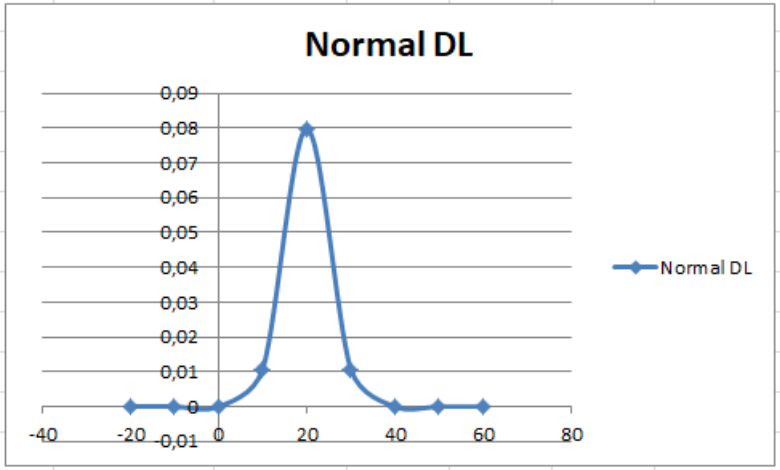
x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15

Solution in Microsoft Excel:

Plot the graph of f(x):

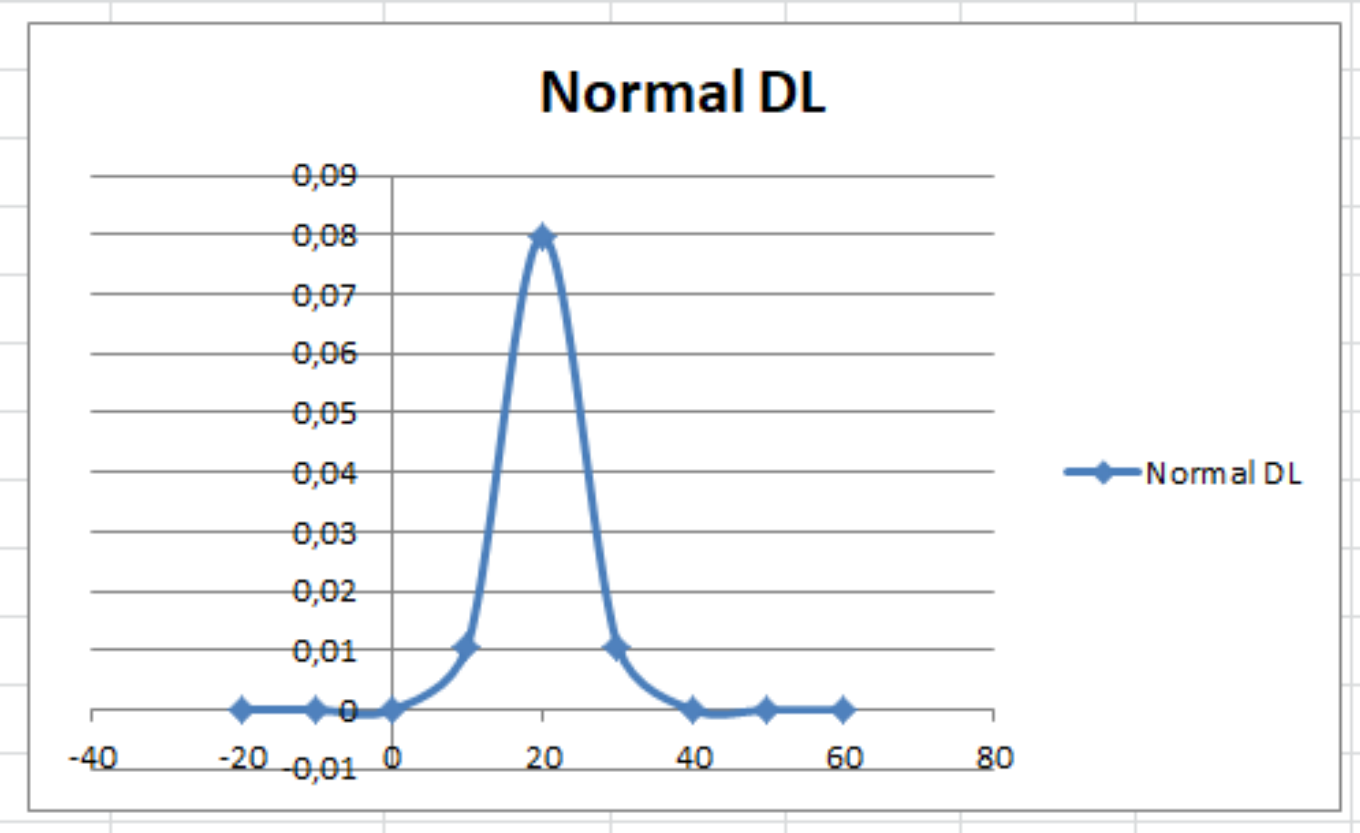
$$f(x) = \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}}$$

x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15



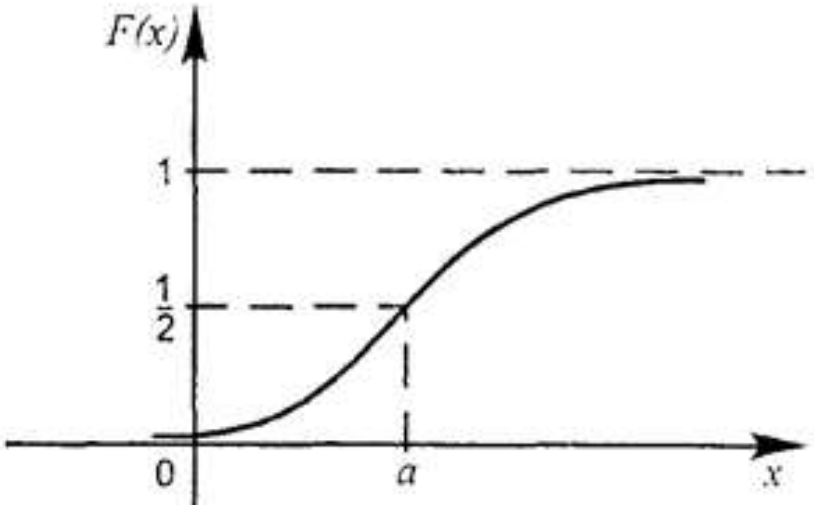
Solution in Microsoft Excel:

Plot the graph of f(x):



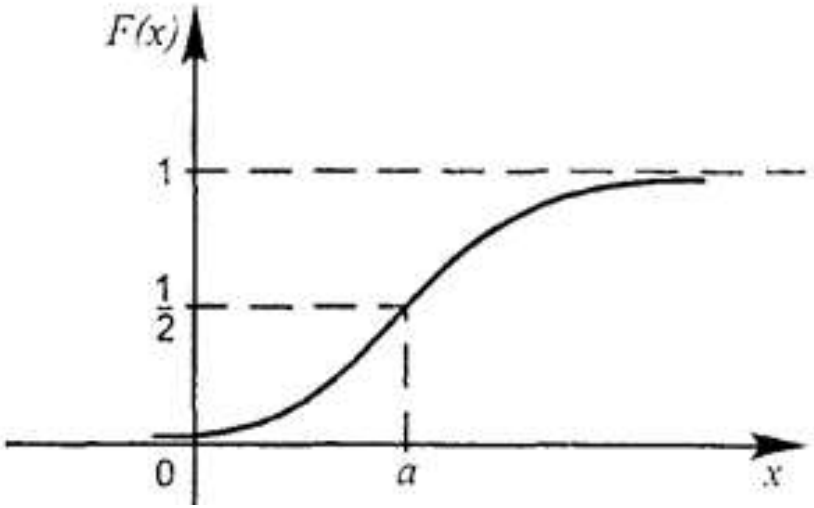
The integral distribution function of the normal law is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}} dx$$



The integral distribution function of the normal law is

$$F(x) = \int_{-\infty}^{x} \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}} dx$$



Solution in Microsoft Excel:

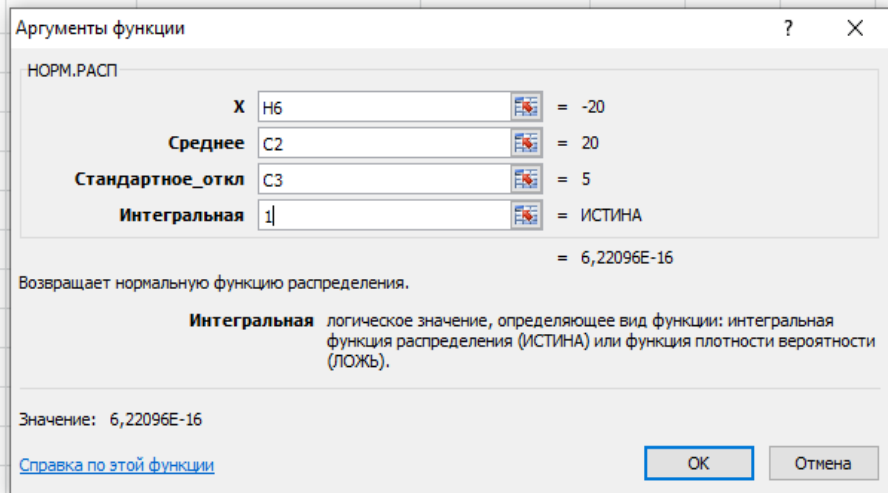
Plot the graph of F(x):

	A	B	C
1	Task 1	Normal DL	
2		a	20
3		σ	5
4		α	13
5		β	24
6		M(x)=a	20
7		D(x)=σ²	25
8		σ(x)=σ	5
9		ν=σ(x)/M(x)*100%	25

$$f(x) = \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}}$$

$$F(x) = \int_{-\infty}^{x}$$

x	-20	-10	0	10	20
f(x)	1,01045E-15	1,2152E-09	3E-05	0,011	0,08
=НОРМ.РАСП(H6;C2;C3;1)					



Solution in Microsoft Excel:

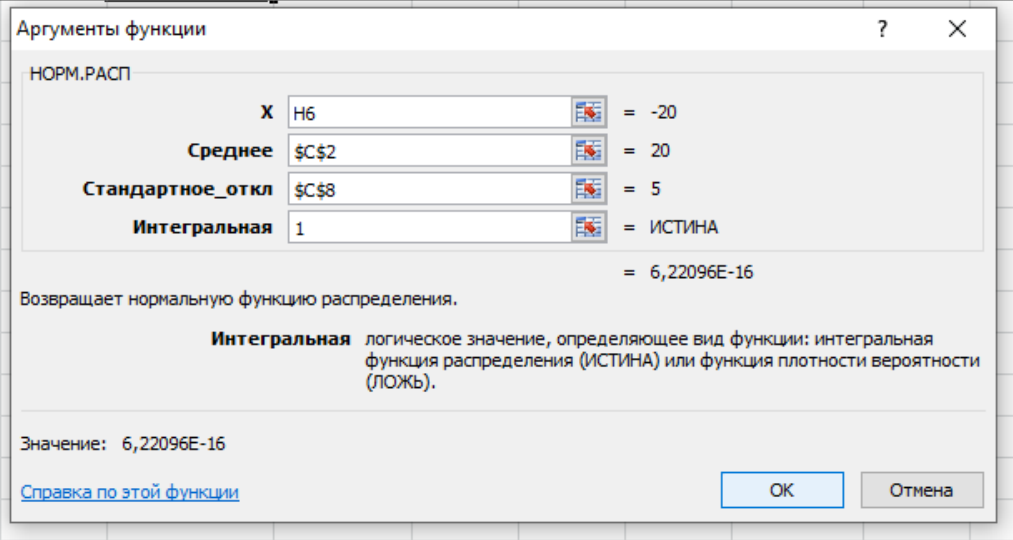
Plot the graph of F(x):

$$f(x)=\frac{1}{5\sqrt{2\pi}}\cdot e^{-\frac{(x-20)^2}{50}}$$

$$F(x)=\int_{-\infty}^{x}\frac{1}{5\sqrt{2\pi}}\cdot e^{-\frac{(x-20)^2}{50}}dx$$

x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15

=НОРМ.РАСП(H6;C2;C8;1)

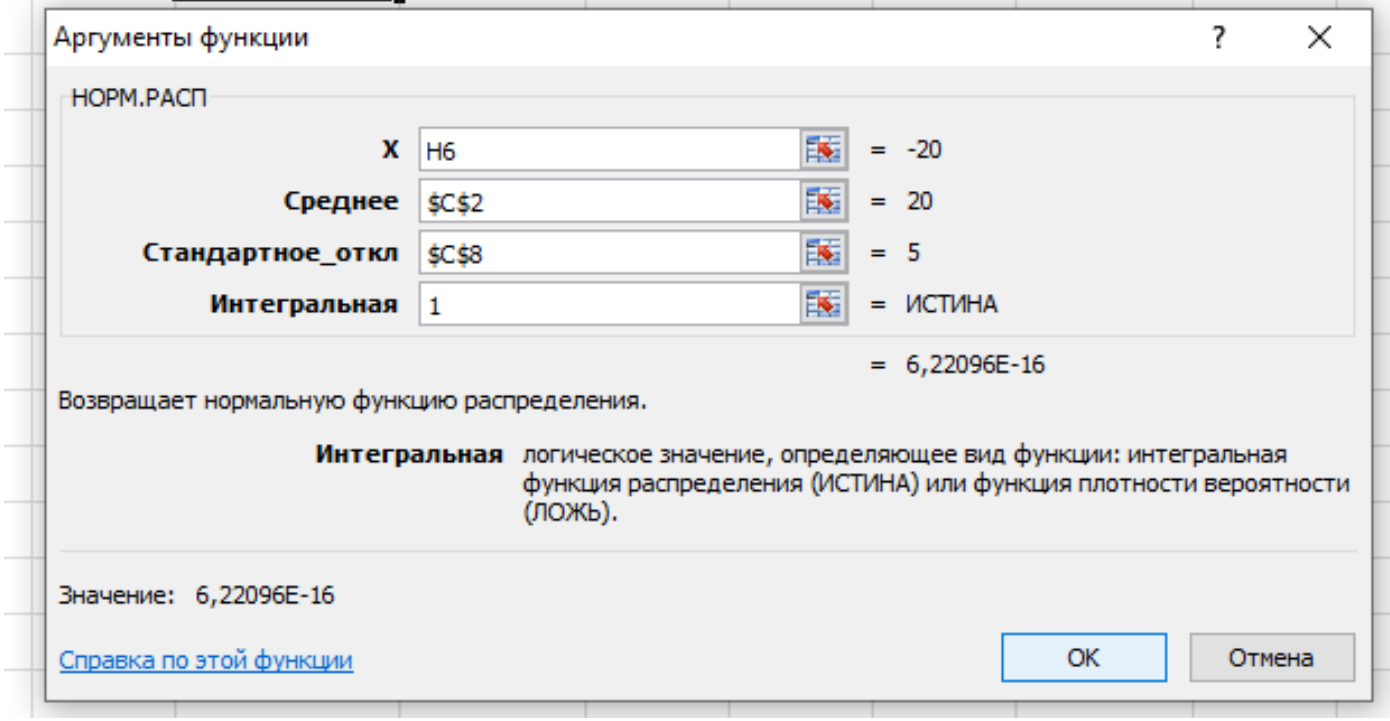


Solution in Microsoft Excel:

Plot the graph of F(x):

x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15

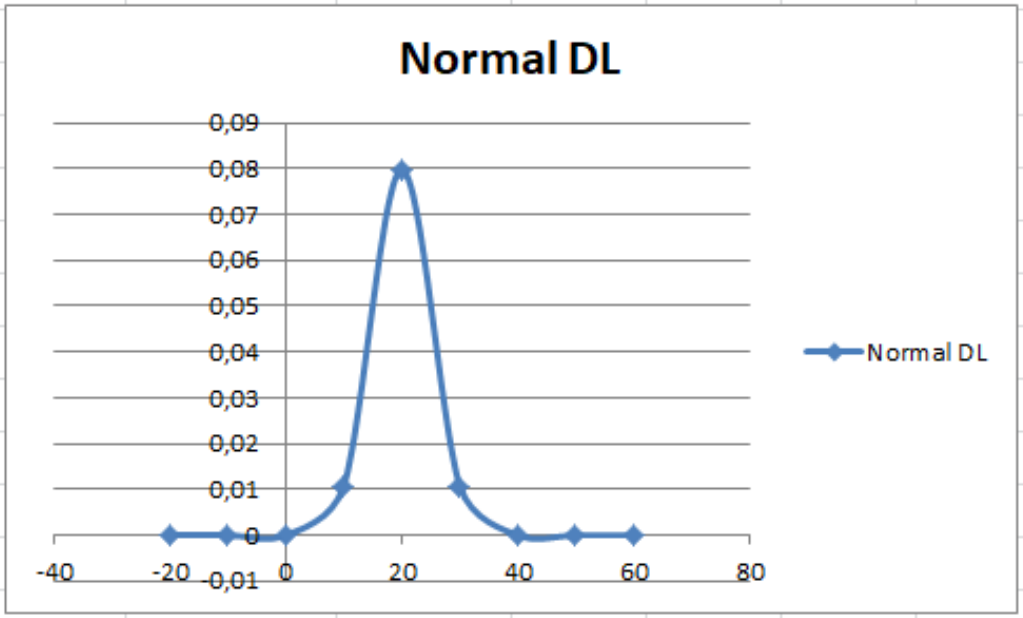
=НОРМ.РАСП(H6;C2;C8;1)



Solution in Microsoft Excel:

Plot the graph of F(x):

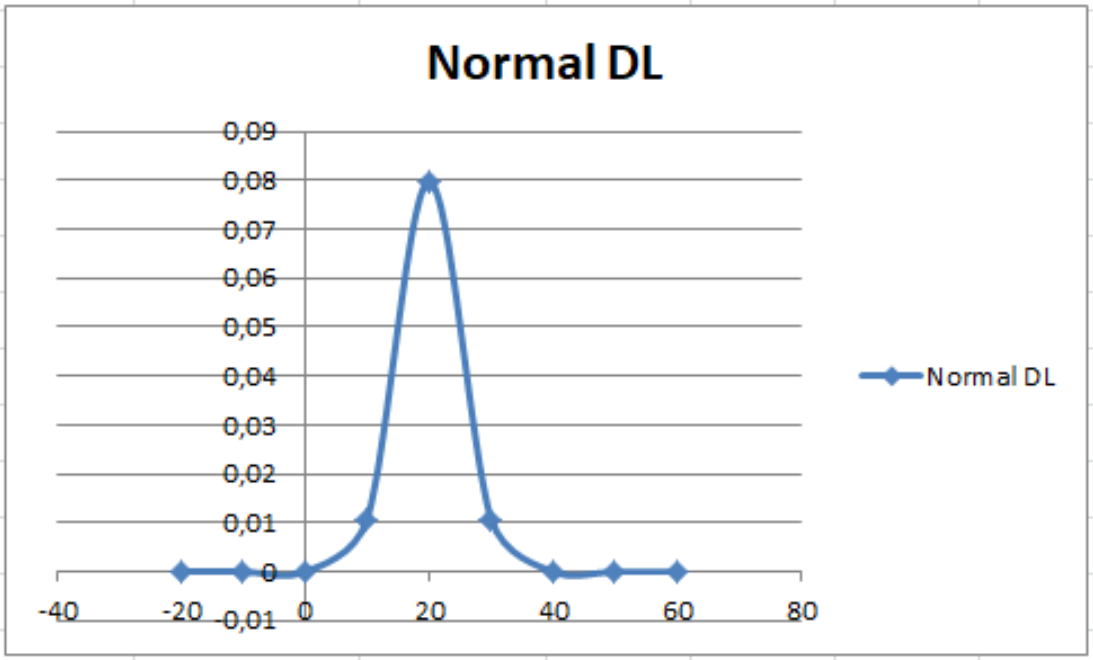
x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15
F(x)	6,22E-16								



Solution in Microsoft Excel:

Plot the graph of F(x):

x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15
F(x)	6,22E-16	1E-09	3E-05	0,023	0,5	0,9772	1	1	1



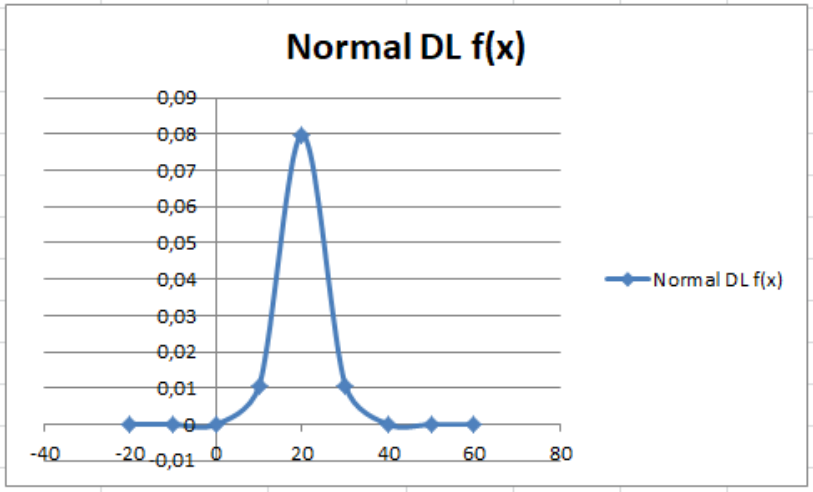
Solution in Microsoft Excel:

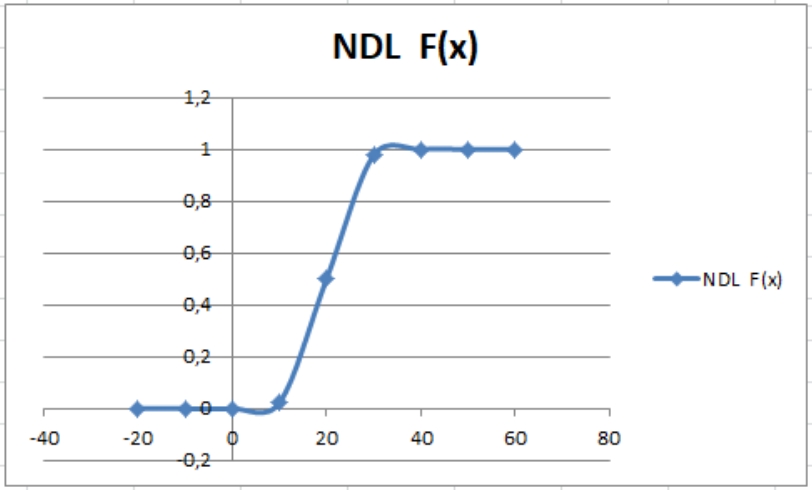
Plot the graph of F(x):

$$f(x) = \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{5\sqrt{2\pi}} \cdot e^{-\frac{(x-20)^2}{50}} dx$$

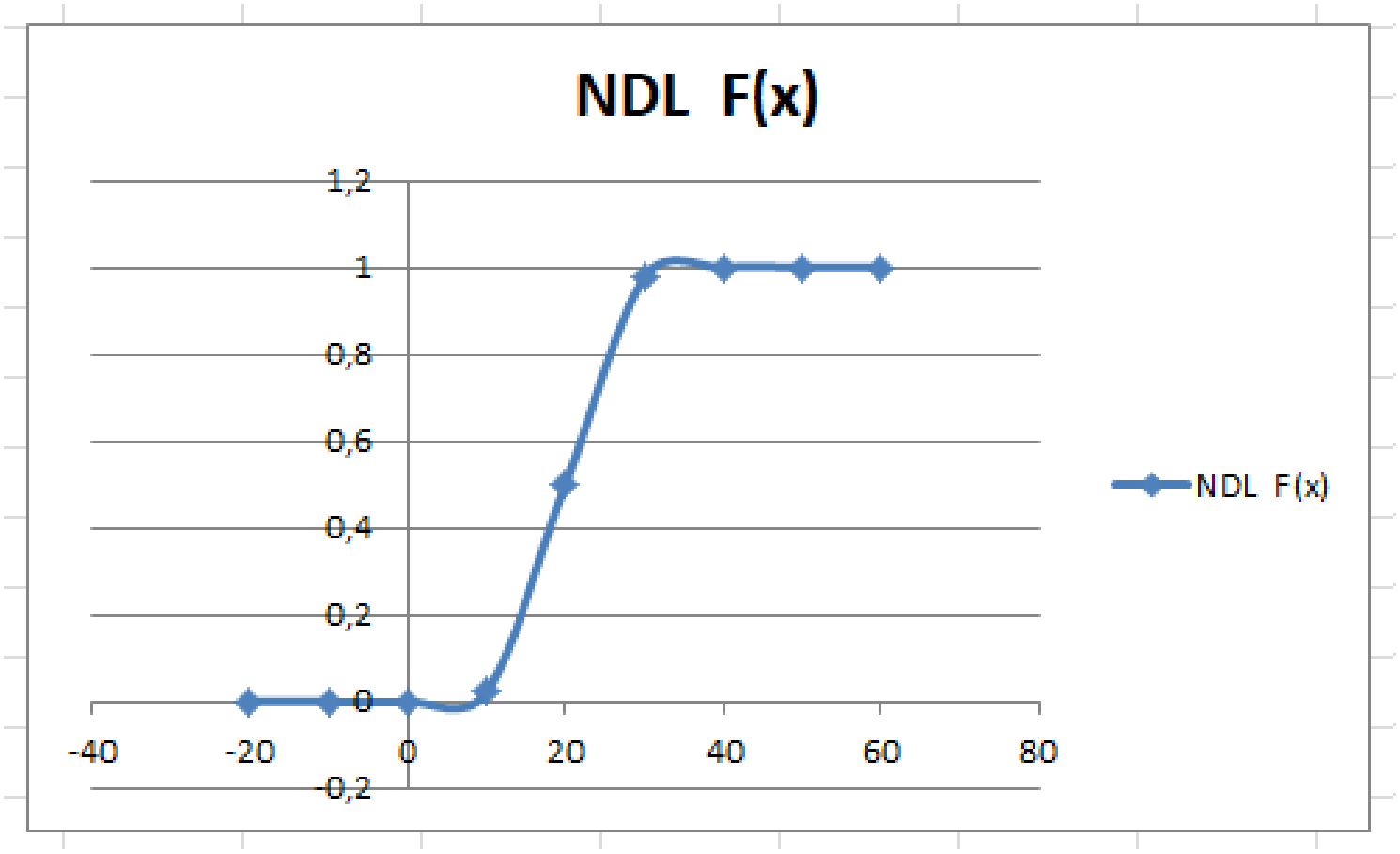
x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01E-15	1,2E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15
F(x)	6,22E-16	1E-09	3E-05	0,023	0,5	0,9772	1	1	1





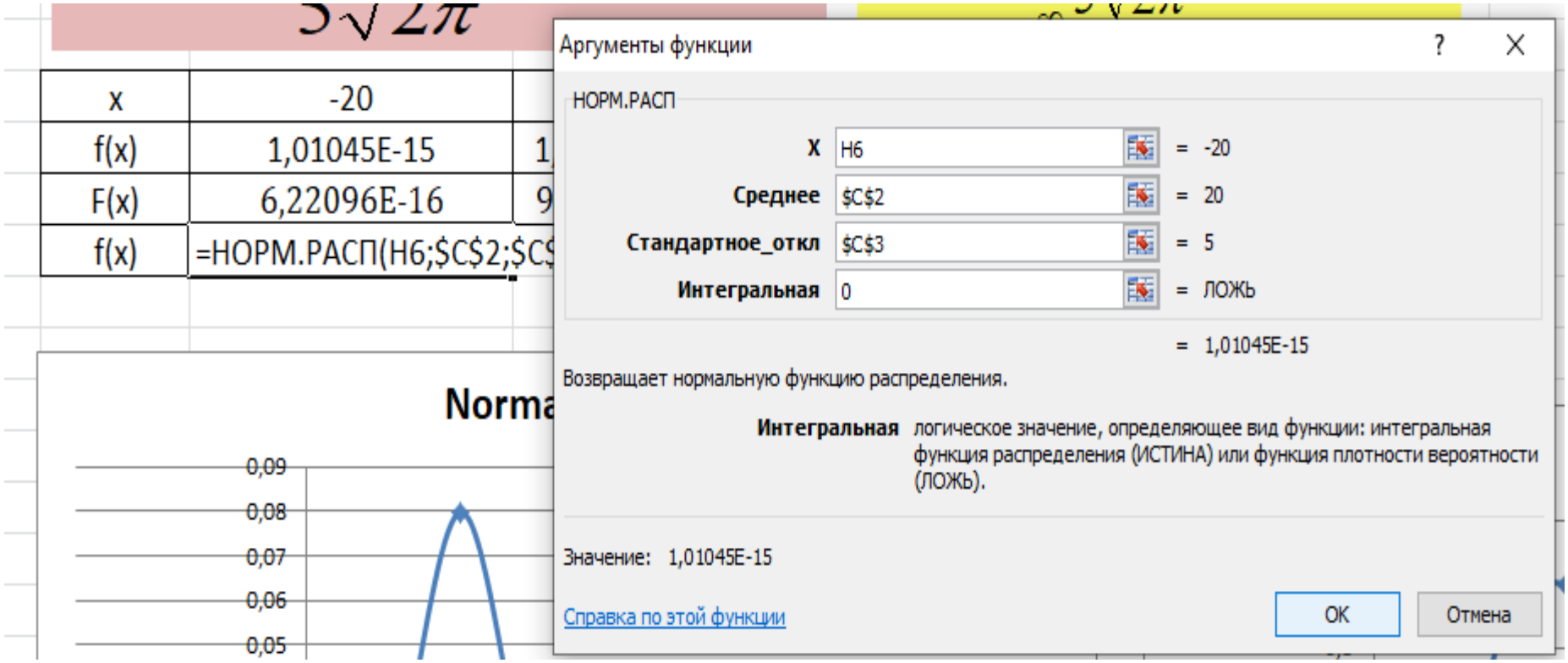
Solution in Microsoft Excel:

Plot the graph of F(x):



Solution in Microsoft Excel:

Checking:



Solution in Microsoft Excel:

Checking:

x	-20	-10	0	10	20	30	40	50	60
f(x)	1,01045E-15	1,2152E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15
F(x)	6,22096E-16	9,866E-10	3E-05	0,023	0,5	0,9772	1	1	1
f(x)	1,01045E-15	1,2152E-09	3E-05	0,011	0,08	0,0108	3E-05	1E-09	1E-15

Solution in Microsoft Excel:

Checking:

0 for f(x) (false or ЛОЖЬ)
1 for F(x) (true or ИСТИНА)

The probability that a random variable X lies in the interval (α, β)

$$P(\alpha < X < \beta) = F(\beta) - F(\alpha) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

a	20
σ	5
α	13
β	24

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1	
(β-a)/σ	=(C27-C24)/C25

Вертикальна

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

a	20
σ	5
α	13
β	24

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1	
(β-a)/σ	0,8
(α-a)/σ	=(C26-C24)/C25

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

a	20
σ	5
α	13
β	24

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1	
(β-a)/σ	0,8
(α-a)/σ	-1,4

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1	
(β-a)/σ	0,8
(α-a)/σ	-1,4
Φ((β-a)/σ)	=Φ(0,8) =

Laplace integral function table $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$ (TABLE 2

x	0	1	2	3	4	5	6	7	8	9
0,0	0,0000	0,0040	0,0080	0,0120	0,0160	0,0199	0,0239	0,0279	0,0319	0,0359
0,1	0,0398	0,0438	0,0478	0,0517	0,0557	0,0596	0,0636	0,0675	0,0714	0,0754
0,2	0,0793	0,0832	0,0871	0,0910	0,0948	0,0987	0,1026	0,1064	0,1103	0,1141
0,3	0,1179	0,1217	0,1255	0,1293	0,1331	0,1368	0,1406	0,1443	0,1480	0,1517
0,4	0,1554	0,1591	0,1628	0,1664	0,1700	0,1736	0,1772	0,1808	0,1844	0,1879
0,5	0,1915	0,1950	0,1985	0,2019	0,2054	0,2088	0,2123	0,2157	0,2190	0,2224
0,6	0,2258	0,2291	0,2324	0,2356	0,2389	0,2422	0,2454	0,2486	0,2518	0,2549
0,7	0,2580	0,2612	0,2642	0,2673	0,2704	0,2734	0,2764	0,2794	0,2823	0,2852
0,8	0,2881	0,2910	0,2939	0,2967	0,2996	0,3023	0,3051	0,3078	0,3106	0,3133
0,9	0,3159	0,3186	0,3212	0,3238	0,3264	0,3289	0,3315	0,3340	0,3365	0,3389

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,8) =	0,2881

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	

Laplace integral function table $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x e^{-\frac{x^2}{2}}dx$ (TABLE 2)

x	0	1	2	3	4	5	6	7	8	9
0,0	0,0000	0,0040	0,0080	0,0120	0,0160	0,0199	0,0239	0,0279	0,0319	0,0359
0,1	0,0398	0,0438	0,0478	0,0517	0,0557	0,0596	0,0636	0,0675	0,0714	0,0754
0,2	0,0793	0,0832	0,0871	0,0910	0,0948	0,0987	0,1026	0,1064	0,1103	0,1141
0,3	0,1179	0,1217	0,1255	0,1293	0,1331	0,1368	0,1406	0,1443	0,1480	0,1517
0,4	0,1554	0,1591	0,1628	0,1664	0,1700	0,1736	0,1772	0,1808	0,1844	0,1879
0,5	0,1915	0,1950	0,1985	0,2019	0,2054	0,2088	0,2123	0,2157	0,2190	0,2224
0,6	0,2258	0,2291	0,2324	0,2356	0,2389	0,2422	0,2454	0,2486	0,2518	0,2549
0,7	0,2580	0,2612	0,2642	0,2673	0,2704	0,2734	0,2764	0,2794	0,2823	0,2852
0,8	0,2881	0,2910	0,2939	0,2967	0,2996	0,3023	0,3051	0,3078	0,3106	0,3133
0,9	0,3159	0,3186	0,3212	0,3238	0,3264	0,3289	0,3315	0,3340	0,3365	0,3389
1,0	0,3413	0,3438	0,3461	0,3485	0,3508	0,3531	0,3554	0,3577	0,3599	0,3621
1,1	0,3643	0,3665	0,3686	0,3708	0,3729	0,3749	0,3770	0,3790	0,3810	0,3830
1,2	0,3849	0,3869	0,3888	0,3906	0,3925	0,3944	0,3962	0,3980	0,3997	0,4015
1,3	0,4032	0,1049	0,4066	0,4082	0,4099	0,4115	0,4131	0,4147	0,4162	0,4177
1,4	0,4192	0,4207	0,4222	0,4236	0,4251	0,4265	0,4274	0,4292	0,4306	0,4319

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		=I30-I31

Solution in Microsoft Excel:

WAY 1. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073

Let's use the relationship:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}} dx = \frac{1}{2} + \Phi(t)$$

$$\Phi\left(\frac{x-a}{\sigma}\right) = F(x) - \frac{1}{2} = F(x) - 0{,}5$$

Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073
way 2		
Φ((β-a)/σ)	=НОРМСТРАСП(H28)-0,5	

Solution in Microsoft Excel:

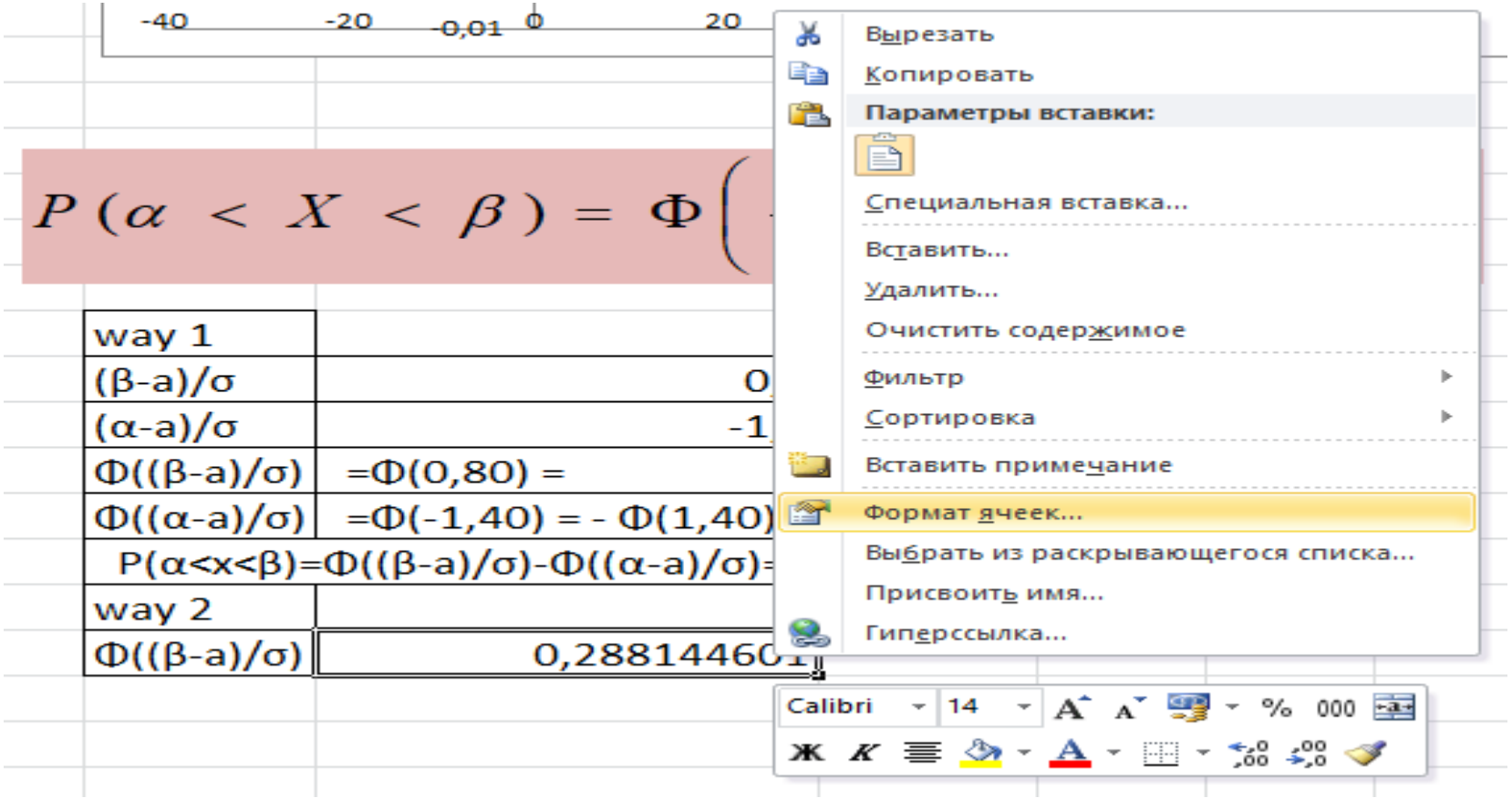
WAY 2. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073
way 2		
Φ((β-a)/σ)	0,288144601	

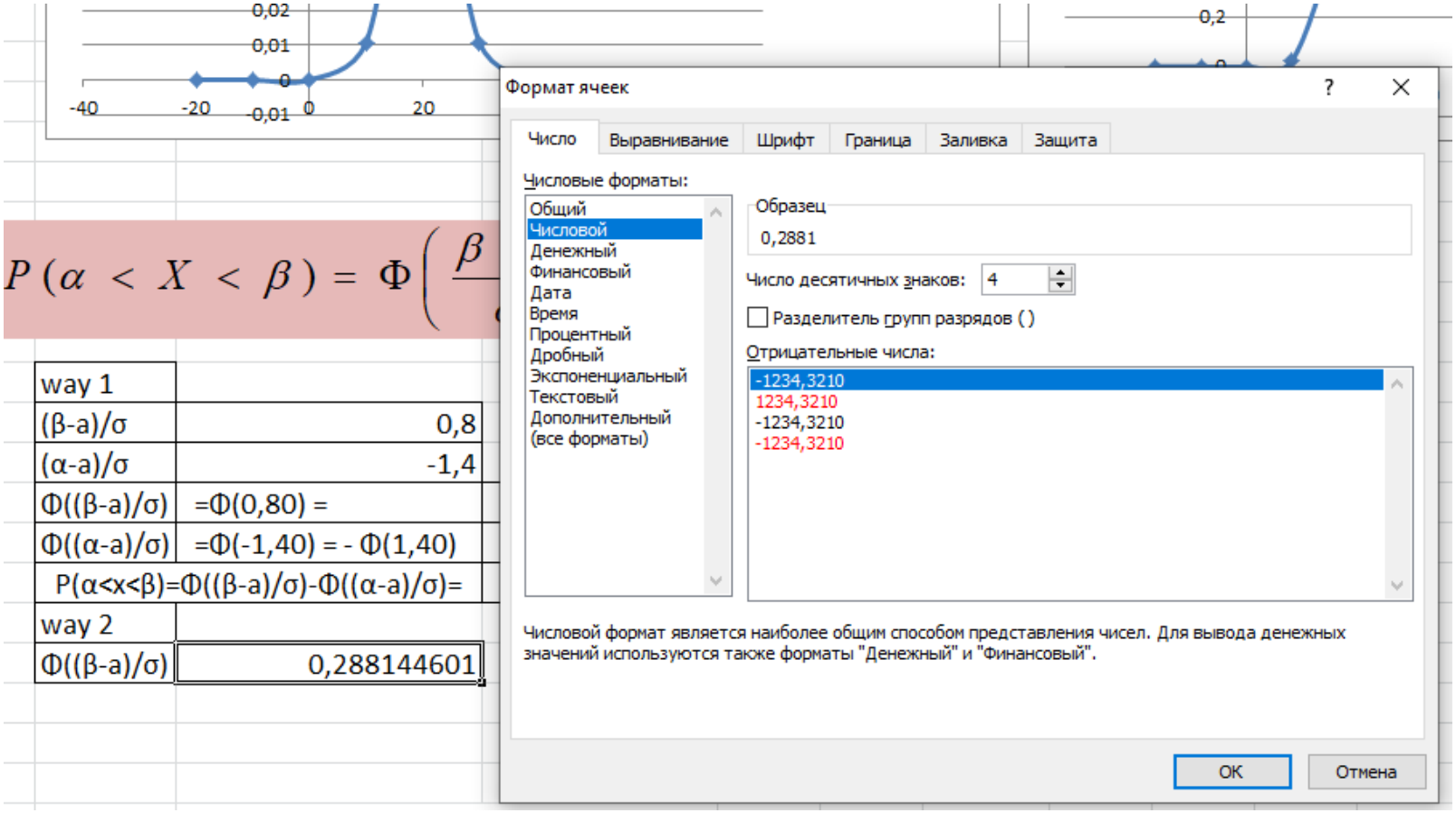
Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)



Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)



way 1	
(β-a)/σ	0,8
(α-a)/σ	-1,4
Φ((β-a)/σ)	=Φ(0,80) =
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=	
way 2	
Φ((β-a)/σ)	0,288144601

Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073
way 2		
Φ((β-a)/σ)	0,2881	

Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073
way 2		
Φ((β-a)/σ)	0,2881446	
Φ((α-a)/σ)	=НОРМСТРАСП(H29)-0,5	

Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073
way 2		
Φ((β-a)/σ)	0,2881	
Φ((α-a)/σ)	-0,4192	

Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073
way 2		
Φ((β-a)/σ)	0,2881	
Φ((α-a)/σ)	-0,4192	
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		=H34-H35

Solution in Microsoft Excel:

WAY 2. Calculate the probability P(13<x<24)

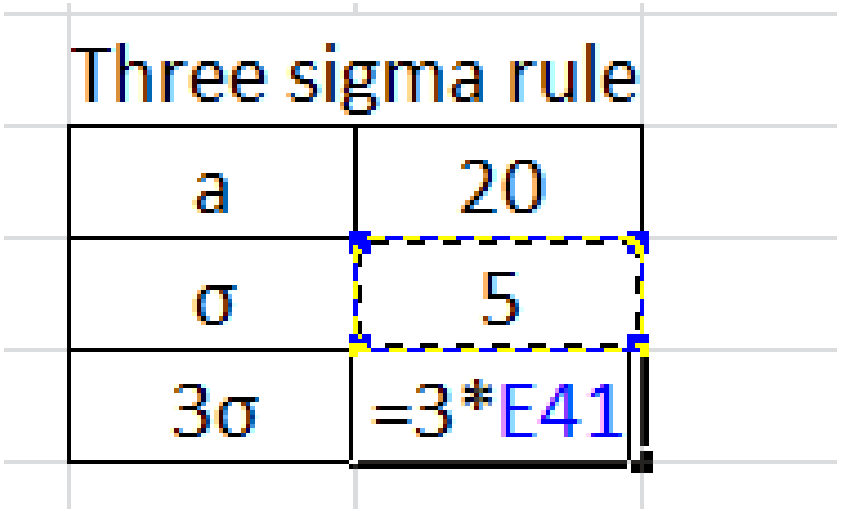
$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

way 1		
(β-a)/σ	0,8	
(α-a)/σ	-1,4	
Φ((β-a)/σ)	=Φ(0,80) =	0,2881
Φ((α-a)/σ)	=Φ(-1,40) = - Φ(1,40)	-0,4192
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073
way 2		
Φ((β-a)/σ)	0,2881446	
Φ((α-a)/σ)	-0,4192433	
P(α<x<β)=Φ((β-a)/σ)-Φ((α-a)/σ)=		0,7073879

Three sigma rule means the normal distributed random variable X possesses all its values on the interval $(a \pm 3\sigma)$ with the probability 99,73% (approximately equals 100 %).

Solution in Microsoft Excel:

Calculate limits of this interval $a - 3\sigma < x < a + 3\sigma$



Solution in Microsoft Excel:

Calculate limits of this interval $a - 3\sigma < x < a + 3\sigma$

Three sigma rule

a	20
σ	5
3σ	15

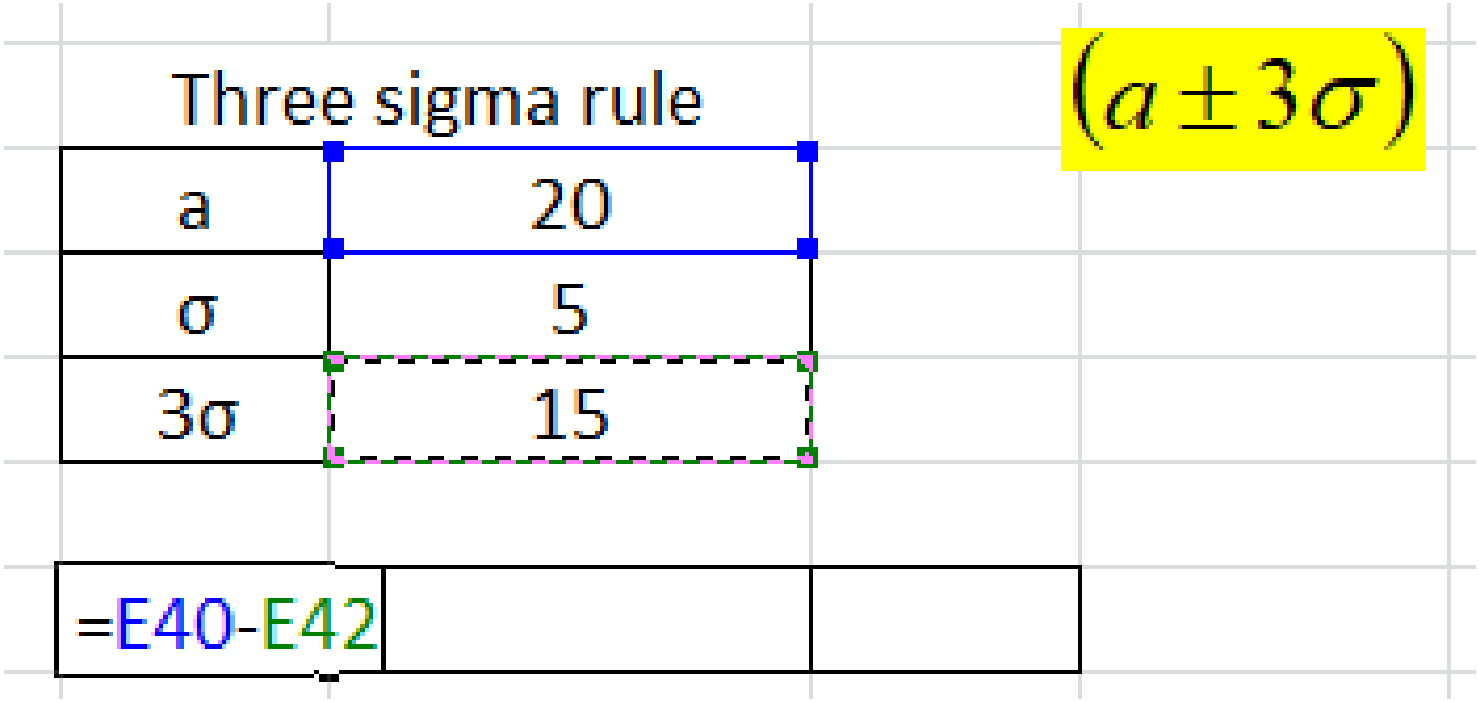
Solution in Microsoft Excel:

Calculate limits of this interval $a - 3\sigma < x < a + 3\sigma$

Three sigma rule	
a	20
σ	5
3σ	15
	< x <	

Solution in Microsoft Excel:

Calculate limits of this interval $a - 3\sigma < x < a + 3\sigma$



Solution in Microsoft Excel:

Calculate limits of this interval $a - 3\sigma < x < a + 3\sigma$

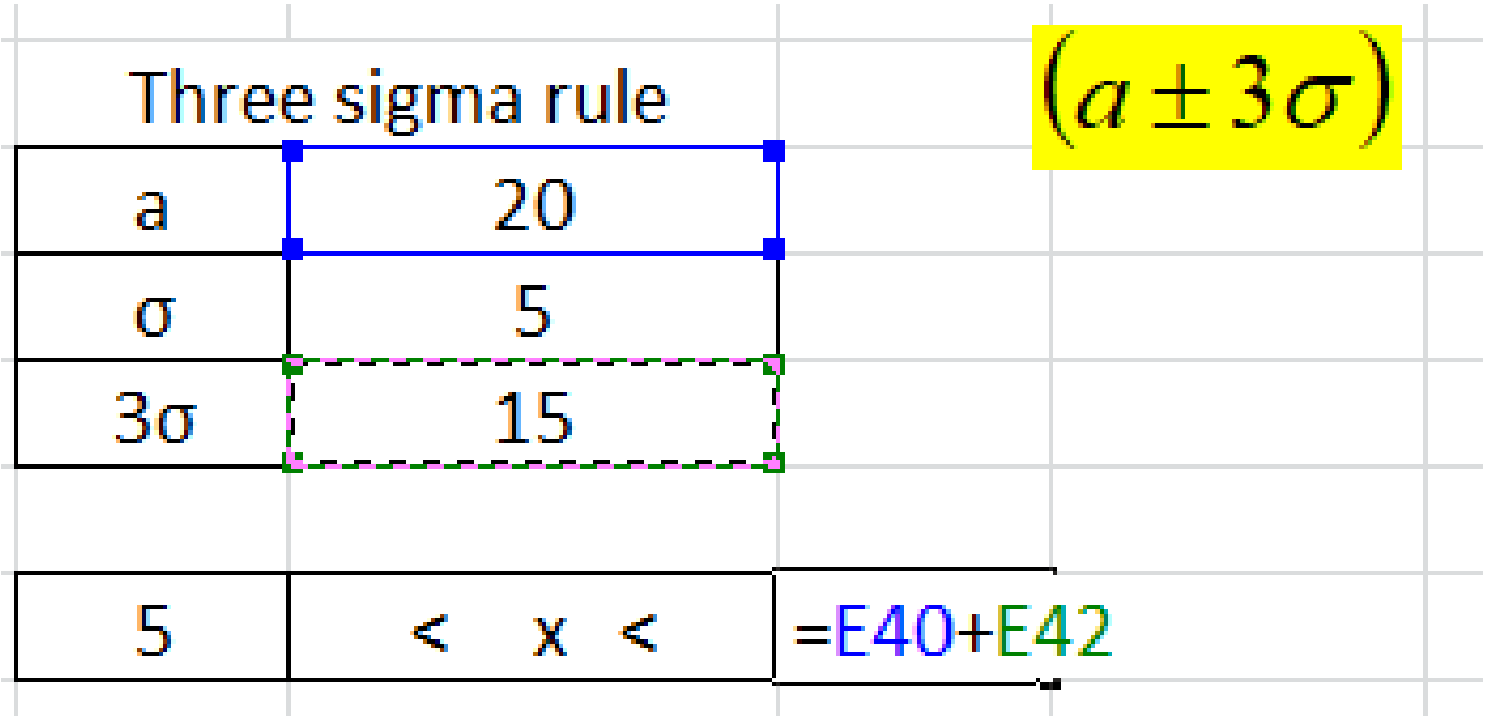
Three sigma rule	
a	20
σ	5
3σ	15

$(a \pm 3\sigma)$

5	< x <	

Solution in Microsoft Excel:

Calculate limits of this interval $a - 3\sigma < x < a + 3\sigma$



Solution in Microsoft Excel:

Calculate limits of this interval $a - 3\sigma < x < a + 3\sigma$

Three sigma rule			
a	20		
σ	5		
3σ	15		
5	< x <	35	

$(a \pm 3\sigma)$