

Theme: GAME THEORY

Task 1.

- 1) Determine the lower and the upper prices of the game and existence of a saddle point for given payoff matrices.
- 2) Solve matrix games of size 2x2 using the analytical method.

(1)	(2)	(3)	(4)	(5)
$A = \begin{pmatrix} 10 & 7 \\ 8 & 11 \end{pmatrix}$	$A = \begin{pmatrix} 4 & -2 \\ 1 & 3 \end{pmatrix}$	$A = \begin{pmatrix} 2 & 8 \\ 7 & 4 \end{pmatrix}$	$A = \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix}$	$A = \begin{pmatrix} 4 & 0 \\ 1 & 3 \end{pmatrix}$
(6)				
$A = \begin{pmatrix} 2 & 9 \\ 8 & 0 \end{pmatrix}$				

Task 2.

- 1) Determine the lower and the upper prices of the game and existence of a saddle point for given payoff matrices.
- 2) Simplify these matrices and exclude unprofitable strategies.

(1)	(2)	(4)
$\begin{pmatrix} 4 & 5 & 6 & 7 & 9 \\ 3 & 4 & 6 & 7 & 6 \\ 7 & 6 & 10 & 8 & 11 \\ 8 & 5 & 4 & 7 & 3 \end{pmatrix}$	$\begin{pmatrix} 3 & -2 & 5 & -1 \\ 4 & 0 & 6 & 1 \\ 2 & -1 & 3 & 2 \\ 1 & 3 & 7 & 4 \end{pmatrix}$	$A = \begin{pmatrix} 3 & 9 & 2 & 1 \\ 7 & 8 & 5 & 6 \\ 4 & 7 & 3 & 5 \\ 5 & 6 & 1 & 7 \end{pmatrix}$
(5)	(6)	(7)
$A = \begin{pmatrix} 4 & 5 & 9 & 3 \\ 8 & 4 & 3 & 7 \\ 7 & 6 & 8 & 9 \\ 7 & 2 & 4 & 6 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 5 & 9 \\ -2 & -4 & -2 & 7 \\ 7 & 5 & 0 & -3 \\ -1 & 6 & 1 & 2 \\ 6 & 9 & 6 & 3 \end{pmatrix}$	$\begin{pmatrix} 6 & 5 & 9 \\ -2 & -2 & 7 \\ 7 & 0 & -3 \end{pmatrix}$
(8)	(10)	(9)
$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 4 \\ 3 & 3 & 2 & 2 \\ 4 & 1 & 3 & 3 \end{pmatrix}$	$\begin{pmatrix} 6 & 7 & 7 & 5 \\ 8 & 1 & 6 & 7 \\ 4 & -3 & 4 & 3 \end{pmatrix}$	$A = \begin{pmatrix} 2 & 4 & 8 & 5 \\ 6 & 2 & 4 & 6 \\ 3 & 2 & 5 & 4 \end{pmatrix}$

Simplify matrices, calculate the lower price and the upper price and find the optimal solution of the game and the game price:

$$\text{Task 3 } A = \begin{pmatrix} 3 & 1 & 4 & 5 \\ 4 & 3 & 5 & 7 \\ 2 & 1 & 3 & 3 \\ 5 & 5 & 4 & 3 \end{pmatrix};$$

$$\text{Task 4 } A = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 4 & 3 & 4 & 5 \\ 3 & 2 & 1 & 4 \\ 5 & 4 & 1 & 6 \end{pmatrix};$$

$$\text{Task 5 } A = \begin{pmatrix} 4 & 1 & 3 & 2 \\ 4 & 2 & 4 & 3 \\ 5 & 2 & 4 & 4 \\ 1 & 4 & 7 & 5 \end{pmatrix};$$

$$\text{Task 6 } A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 2 & 3 & 3 \\ 0 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 \end{pmatrix};$$

$$\text{Task 7 } A = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 2 & 3 & 3 & 4 \\ 0 & 2 & 1 & 3 \\ 5 & 2 & 3 & 5 \end{pmatrix}.$$

$$\text{Task 8 } \Pi = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 3 & 2 & 1 & 2 \\ 0 & 4 & 2 & 3 \\ 5 & 3 & 1 & 4 \end{pmatrix}$$

$$\text{Task 9 } \Pi = \begin{pmatrix} -1 & 7 & 5 & 6 & 12 \\ 4 & 8 & 9 & 6 & 15 \\ 5 & -7 & 4 & 5 & 14 \end{pmatrix}$$

$$\text{Task 10 } \Pi = \begin{pmatrix} 4 & 1 & -2 & 3 & 5 \\ 6 & 3 & 1 & 3 & 7 \\ 3 & 2 & 6 & 4 & 4 \end{pmatrix}$$

$$\text{Task 11 } \Pi = \begin{pmatrix} 5 & 2 & 1 & 3 \\ 6 & 3 & 2 & 2 \\ 2 & 4 & 0 & -3 \end{pmatrix}$$

$$\text{Task 12 } \Pi = \begin{pmatrix} 1 & 2 & 4 & 3 \\ 0 & 2 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 4 & 3 & 1 & 0 \end{pmatrix}$$

$$\text{Task 13 } \Pi = \begin{pmatrix} 2 & 0 & 1 & 4 \\ 1 & 2 & 5 & 3 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

The payoff matrices are given. Solve the matrix game by the graphical method, find the optimal strategies and the game price.

$$\text{Task 14 } \Pi = \begin{pmatrix} 7 & 4 & 1 & 7 & -2 \\ 5 & 0 & 4 & -3 & 2 \end{pmatrix}$$

$$\text{Task 15 } \Pi = \begin{pmatrix} 4 & 7 & 6 & 5 & 2 \\ 6 & 3 & 7 & 4 & 8 \end{pmatrix}^T$$

$$\text{Task 16 } \Pi = \begin{pmatrix} 5 & 4 & 6 & 4 & 6 \\ 4 & 1 & 8 & 4 & 2 \end{pmatrix}$$

$$\text{Task 17 } \Pi = \begin{pmatrix} 1 & 5 & 9 & 3 \\ 6 & 3 & 2 & 7 \end{pmatrix}$$

$$\text{Task 18 } \Pi = \begin{pmatrix} 11 & 9 & 6 & 0 \\ 2 & 6 & 8 & 10 \end{pmatrix}^T$$

$$\text{Task 19 } \Pi = \begin{pmatrix} 0 & 5 & 3 & 4 \\ 5 & 3 & 5 & 4 \end{pmatrix}$$

$$\text{Task 20 } \Pi = \begin{pmatrix} 2 & 3 & 6 & 5 \\ 6 & 4 & 1 & 3 \end{pmatrix}$$

$$\text{Task 21 } \Pi = \begin{pmatrix} 7 & 2 & 6 & 4 \\ 2 & 6 & 5 & 7 \end{pmatrix}^T$$

$$\text{Task 22 } \Pi = \begin{pmatrix} 2 & 4 & 5 & 7 \\ 8 & 6 & 4 & 3 \end{pmatrix}^T$$

$$\text{Task 23 } \Pi = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 7 & 1 & 2 \end{pmatrix}$$

$$\text{Task 24 } \Pi = \begin{pmatrix} 4 & 6 & 8 & 7 \\ 6 & 3 & 2 & 5 \end{pmatrix}^T$$

$$\text{Task 25 } \Pi = \begin{pmatrix} 3 & 2 & 1 & 1 \\ 1 & 3 & 4 & 2 \end{pmatrix}^T$$

$$\text{Task 26 } \Pi = \begin{pmatrix} 6 & 5 & 2 & 4 \\ -1 & 1 & 4 & 2 \end{pmatrix}^T$$