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Theme 8(part 1). The elements of correlation theory

Example 2

Y \ X	2	4	6	8	m_y	$\overline{x_{y_i}}$
1	1	2				
3	1	6	3			
5		5	5			
7			1	1		
m_x						
$\overline{y_{x_j}}$						

Example 2

Y \ X	2	4	6	8	m_y	$\overline{x}_{y_i}$
1	1	2			3	
3	1	6	3		10	
5		5	5		10	
7			1	1	2	
m_x	2	13	9	1	25	
$\overline{y}_{x_j}$						

For example, at $x = 2$ the variable **y** takes values: $y_1 = 1$, $y_2 = 3$.

Then arithmetic mean **Y**, which corresponds to $x = 2$:

$$\overline{y}_{x=2} = \frac{1 \cdot 1 + 3 \cdot 1}{2} = \frac{4}{2} = 2$$

For example, at $x=4$ the variable **y** takes values: $y_1=1$, $y_2=3$, $y_3=5$.

Then arithmetic mean **Y**, which corresponds to $x=4$:

$$\overline{y}_{x=4}=\frac{1\cdot 2+3\cdot 6+5\cdot 5}{13}=\frac{45}{13}=3{,}5$$

Y \ X	2	4	6	8	m_y	$\overline{x_{y_i}}$
1	1	2				
3	1	6	3			
5		5	5			
7			1	1		
m_x	**2**	**13**	**9**	**1**	**25**	
$\overline{y_{x_j}}$	**2**	**3,5**	**4,6**	**9**		

For example, at $y=1$ the variable **y** takes values: $x_1 = 2, \; x_2 = 4$.

Then arithmetic mean **Y**, which corresponds to $y=1$:

$$\bar{x}_{y=1} = \frac{2 \cdot 1 + 4 \cdot 2}{3} = \frac{10}{3} = 3{,}3$$

Example 2

Let's get back to the correlation table:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3			
3	1	6	3		10			
5		5	5		10			
7			1	1	2			
m_{x_i}	2	13	9	1	25			
$x_i m_{x_i}$								
$x_i^2 m_{x_i}$								
$x_i y_j m_{ij}$								

Example 2

Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3			
3	1	6	3		10			
5		5	5		10			
7			1	1	2			
m_{x_i}	2	13	9	1	25			
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$								
$x_i y_j m_{ij}$								

Example 2

Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3			
3	1	6	3		10			
5		5	5		10			
7			1	1	2			
m_{x_i}	2	13	9	1	25			
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$								
$x_i y_j m_{ij}$								

$$\bar{x} = \frac{2 \cdot 2 + 4 \cdot 13 + 6 \cdot 9 + 8 \cdot 1}{25} = \frac{118}{25} = 4{,}72$$

Example 2

Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3	3		
3	1	6	3		10	30		
5		5	5		10	50		
7			1	1	2	14		
m_{x_i}	2	13	9	1	25	97		
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$								
$x_i y_j m_{ij}$								

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Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3	3		
3	1	6	3		10	30		
5		5	5		10	50		
7			1	1	2	14		
m_{x_i}	2	13	9	1	25	97		
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$								
$x_i y_j m_{ij}$								

$$\overline{y} = \frac{1 \cdot 3 + 3 \cdot 10 + 5 \cdot 10 + 7 \cdot 2}{25} = \frac{97}{25} = 3{,}88$$

Example 2

Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3	3		
3	1	6	3		10	30		
5		5	5		10	50		
7			1	1	2	14		
m_{x_i}	2	13	9	1	25	97		
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$								

Example 2

Let’s calculate:

Y \ X	2	4	6	8	m_{y_i}	$y_j m_{y_i}$	$y_j^2 m_{y_i}$	$x_i y_j m_{ij}$
1	1	2			3	3		
3	1	6	3		10	30		
5		5	5		10	50		
7			1	1	2	14		
m_{x_i}	2	13	9	1	25	97		
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$								

$$\overline{x^2} = \frac{4 \cdot 2 + 16 \cdot 13 + 36 \cdot 9 + 64 \cdot 1}{25} = \frac{604}{25} = 24{,}16$$

Example 2

Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3	3	3	
3	1	6	3		10	30	90	
5		5	5		10	50	250	
7			1	1	2	14	98	
m_{x_i}	2	13	9	1	25	97	441	
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$								

Example 2

Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1	2			3	3	3	
3	1	6	3		10	30	90	
5		5	5		10	50	250	
7			1	1	2	14	98	
m_{x_i}	2	13	9	1	25	97	441	
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$								

$$\overline{y^2} = \frac{1\cdot 3 + 9\cdot 10 + 25\cdot 10 + 49\cdot 2}{25} = \frac{441}{25} = 17{,}64$$

Example 2

Let's calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1 2	2 8			3	3	3	
3	1 6	6 72	3 54		10	30	90	
5		5 100	5 150		10	50	250	
7			1 42	1 56	2	14	98	
m_{x_i}	2	13	9	1	25	97	441	
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$								

Example 2

Let’s calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1 2	2 8			3	3	3	
3	1 6	6 72	3 54		10	30	90	
5		5 100	5 150		10	50	250	
7			1 42	1 56	2	14	98	
m_{x_i}	2	13	9	1	25	97	441	
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$	8	180	246	56	490			

Example 2

Let’s calculate:

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1 2	2 8			3	3	3	10
3	1 6	6 72	3 54		10	30	90	132
5		5 100	5 150		10	50	250	250
7			1 42	1 56	2	14	98	98
m_{x_i}	2	13	9	1	25	97	441	490
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$	8	180	246	56	490			

Example 2

Y \ X	2	4	6	8	m_{y_j}	$y_j m_{y_j}$	$y_j^2 m_{y_j}$	$x_i y_j m_{ij}$
1	1 2	2 8			3	3	3	10
3	1 6	6 72	3 54		10	30	90	132
5		5 100	5 150		10	50	250	250
7			1 42	1 56	2	14	98	98
m_{x_i}	2	13	9	1	25	97	441	490
$x_i m_{x_i}$	4	52	54	8	118			
$x_i^2 m_{x_i}$	8	208	324	64	604			
$x_i y_j m_{ij}$	8	180	246	56	490			

$$\overline{xy} = \frac{\sum_i \sum_k m_{ki} x_i y_k}{n} = \frac{1}{25}\left(1\cdot 2\cdot 1 + 3\cdot 2\cdot 1 + 1\cdot 4\cdot 2 + 3\cdot 4\cdot 6 + 5\cdot 4\cdot 5 +\right.$$

$$\left. + 3\cdot 6\cdot 3 + 5\cdot 6\cdot 5 + 7\cdot 6\cdot 1 + 7\cdot 8\cdot 1\right) = \frac{1}{25}\left(2+6+8+72+100+54+150+42+56\right) = \frac{490}{25} = 19{,}6$$

Example 2

Let's calculate:

$$\rho_{y/x} = b_1 = \frac{19{,}6 - 4{,}72 \cdot 3{,}88}{24{,}16 - 4{,}72^2} = \frac{1{,}2864}{1{,}8816} \approx 0{,}68$$

Example 2

Let's calculate:

$$\rho_{y/x} = b_1 = \frac{19,6 - 4,72 \cdot 3,88}{24,16 - 4,72^2} = \frac{1,2864}{1,8816} \approx 0,68$$

$$b_0 = 3,88 - 0,68 \cdot 4,72 \approx 0,67$$

Example 2

Let's calculate:

$$\rho_{y/x} = b_1 = \frac{19,6 - 4,72 \cdot 3,88}{24,16 - 4,72^2} = \frac{1,2864}{1,8816} \approx 0,68$$

$$b_0 = 3,88 - 0,68 \cdot 4,72 \approx 0,67$$

$$\tilde{y}_x = 0,68x + 0,67$$

Example 2

The theoretical regression equation is

$$\tilde{y}_x = 0{,}68x + 0{,}67$$

Explanation: the coefficient $b_1 = 0{,}68$ shows the increasing X by 1 unit gives the increasing Y by 0,68 units.

Example 2

The correlation coefficient:

$$r = \frac{\mu_{xy}}{\sigma_x \cdot \sigma_y} = \frac{1{,}2864}{\sqrt{1{,}8816 \cdot 2{,}5856}} \approx 0{,}5832$$

Example 2

The correlation coefficient:

$$r = \frac{\mu_{xy}}{\sigma_x \cdot \sigma_y} = \frac{1{,}2864}{\sqrt{1{,}8816 \cdot 2{,}5856}} \approx 0{,}5832$$

Then this linear correlation is moderate (средняя).

Example. Elasticity

In economics, **elasticity** is the measurement of how responsive an economic variable is to a change in another.

$$\overline{E} = b_1 \cdot \frac{\overline{x}}{\overline{y}} = 0{,}68 \cdot \frac{4{,}72}{3{,}88} = 0{,}83\%$$

Example. Elasticity

Conclusion: The elasticity coefficient (0,83%) is a number that indicates the percentage change that will occur in one variable (*y*) when the variable *x* changes one percent.

$$\overline{E} = b_1 \cdot \frac{\overline{x}}{\overline{y}} = 0{,}68 \cdot \frac{4{,}72}{3{,}88} = 0{,}83\%$$

EXAMPLE

Let’s continue to solve **EXAMPLE 2.**

$$r = 0,5832$$

EXAMPLE

Let’s continue to solve **EXAMPLE 2.**

$$r = 0,5832$$

$$R^2 = r_{xy}^2 = 0,5832^2 = 0,3401$$

EXAMPLE

Let's continue to solve **EXAMPLE 2.**

$$r = 0{,}5832$$

$$R^2 = r_{xy}^2 = 0{,}5832^2 = 0{,}3401$$

It means that 34,01% of the total variation in ***y*** can be explained by the linear relationship between ***x*** and ***y*** (as described by the regression equation). The other 100%-34,01%=65,99% of the total variation in ***y*** remains unexplained.