Doc. Misiura Ie.Iu.

CALCULATION of LIMITS using OCTAVE

Example 1:

$$\lim_{x\to 1}\left(x^2-3x+5\right)=1-3+5=3.$$

USE OCTAVE
https://octave-online.net/

```
octave:1> syms x

octave:2> f=x^2-3*x+5
f = (sym)

   2
  x  - 3·x + 5

octave:3> limit(f,1)
ans = (sym) 3
```

Example 9: Calculate the limit:

$$\lim_{x\to 2}\frac{x^2-4}{x^2-x-2}=\left[\frac{0}{0}\right]=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(x+1)}=\lim_{x\to 2}\frac{x+2}{x+1}=\frac{4}{3}$$

USE OCTAVE
https://octave-online.net/

```
octave:4> f=(x^2-4)/(x^2-x-2)
f = (sym)

    2
   x  - 4
 ──────────
   2
  x  - x - 2

octave:5> limit(f,2)
ans = (sym) 4/3
```

Example 12.1: Calculate the limit:

$$\lim_{x\to\infty}\frac{(x+10)^2}{8x^2+10}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{x^2+20x+100}{8x^2+10}=\lim_{x\to\infty}\frac{1+\frac{20}{x}+\frac{100}{x^2}}{8+\frac{10}{x^2}}=\frac{1}{8}$$

USE OCTAVE

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octave:6> f=(x+10)^2/(8*x^2+10)

f = (sym)

```
        2
(x + 10)
-----------
   2
8·x  + 10
```

octave:7> limit(f,inf)

ans = (sym) 1/8

Example 13:

$$\lim_{x\to\infty}\frac{x^2+2x-3}{x^3+7x+1}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^2}-\frac{3}{x^3}}{1+\frac{7}{x^2}+\frac{1}{x^3}}=\frac{0}{1}=0$$

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octave:8> f=(x^2+2*x-3)/(x^3+7*x+1)

f = (sym)

```
 2
x  + 2·x - 3
--------------
 3
x  + 7·x + 1
```

octave:9> limit(f,inf)

ans = (sym) 0

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Example 14.1: Calculate the limit:

$$\lim_{x\to\infty}\frac{2x^2-3x-5}{x+1}=\frac{\infty}{\infty}=$$

$$=\lim_{x\to\infty}\frac{\frac{2x^2-3x-5}{x^2}}{\frac{x+1}{x^2}}=\lim_{x\to\infty}\frac{2-\frac{3}{x}^{\to 0}-\frac{5}{x^2}^{\to 0}}{\frac{1}{x}^{\to 0}+\frac{1}{x^2}^{\to 0}}=\frac{2}{0}=\infty$$

USE OCTAVE

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octave:10> f=(2*x^2-3*x-5)/(x+1)

f = (sym)

```
   2
2·x  - 3·x - 5
──────────────
    x + 1
```

octave:11> limit(f,inf)

ans = (sym) ∞

Example 17.1:

$$\lim_{x\to 6}\frac{\sqrt{x-2}-2}{x-6}=\left[\frac{0}{0}\right]=\lim_{x\to 6}\frac{(\sqrt{x-2}-2)(\sqrt{x-2}+2)}{(x-6)(\sqrt{x-2}+2)}=$$

$$=\lim_{x\to 6}\frac{x-2-4}{(x-6)(\sqrt{x-2}+2)}=\lim_{x\to 6}\frac{1}{\sqrt{x-2}+2}=\frac{1}{4}$$

USE OCTAVE

https://octave-online.net/

octave:12> f=(sqrt(x-2)-2)/(x-6)

f = (sym)

```
  _______
╲╱ x - 2  - 2
─────────────
    x - 6
```

octave:13> limit(f,6)

ans = (sym) 1/4

Example 17.2:

$$\lim_{x\to 2}\frac{\sqrt{3x-2}-2}{\sqrt{2x+5}-3}=\left(\frac{0}{0}\right)=\lim_{x\to 2}\frac{\left(\sqrt{3x-2}-2\right)\left(\sqrt{3x-2}+2\right)}{\left(\sqrt{2x+5}-3\right)\left(\sqrt{3x-2}+2\right)}=$$

$$=\lim_{x\to 2}\frac{3x-2-4}{\left(\sqrt{2x+5}-3\right)\left(\sqrt{3x-2}+2\right)}=\lim_{x\to 2}\frac{3(x-2)\left(\sqrt{2x+5}+3\right)}{\left(\sqrt{2x+5}-3\right)\left(\sqrt{2x+5}+3\right)\left(\sqrt{3x-2}+2\right)}=$$

$$=\lim_{x\to 2}\frac{3(x-2)\left(\sqrt{2x+5}+3\right)}{(2x+5-9)\left(\sqrt{3x-2}+2\right)}=\lim_{x\to 2}\frac{3(x-2)\left(\sqrt{2x+5}+3\right)}{2(x-2)\left(\sqrt{3x-2}+2\right)}=$$

$$=\frac{3}{2}\lim_{x\to 2}\frac{\sqrt{2x+5}+3}{\sqrt{3x-2}+2}=\frac{3}{2}\cdot\frac{3+3}{2+2}=\frac{18}{8}=\frac{9}{4}$$

USE OCTAVE
https://octave-online.net/

octave:14> f=(sqrt(3*x-2)-2)/(sqrt(2*x+5)-3)
f = (sym)

```
  _________
╲╱ 3·x - 2  - 2
───────────────
  _________
╲╱ 2·x + 5  - 3
```

octave:15> limit(f,2)
ans = (sym) 9/4

Example 19:

$$\lim_{x\to 0}\frac{\sin 5x}{tg\,8x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{5x}{8x}=\frac{5}{8}$$

$$x\to 0,\ \sin 5x\sim 5x,\ tg8x\sim 8x,$$

USE OCTAVE
https://octave-online.net/

octave:16> f=sin(5*x)/tan(8*x)
f = (sym)

```
sin(5·x)
────────
tan(8·x)
```

octave:17> limit(f,0)
ans = (sym) 5/8

Example 20:

$$A=\lim_{x\to\infty}\left(\frac{x+2}{x-4}\right)^{x}=\left\|1^{\infty}\right\|, \text{ because}$$

$$\lim_{x\to\infty}\frac{x+2}{x-4}=\left\|\frac{\infty}{\infty}\right\|=\lim_{x\to\infty}\frac{\frac{x}{x}+\frac{2}{x}}{\frac{x}{x}-\frac{4}{x}}=\lim_{x\to\infty}\frac{1+\frac{2}{x}}{1-\frac{4}{x}}=\frac{1+0}{1-0}=1$$

Then

$$A=\lim_{x\to\infty}\left(1+\frac{x+2}{x-4}-1\right)^{x}=\lim_{x\to\infty}\left(1+\frac{6}{x-4}\right)^{x}=$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-4}{6}}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-4}{6}}\right)^{\frac{x-4}{6}\frac{6}{x-4}x}=$$

$$=\lim_{x\to\infty}\left(\underbrace{\left(1+\frac{1}{\frac{x-4}{6}}\right)^{\frac{x-4}{6}}}_{\to e}\right)^{\frac{6}{x-4}\cdot x}=$$

$$=\lim_{x\to\infty}e^{\frac{6}{x-4}\cdot x}=e^{\lim\limits_{x\to\infty}\frac{6x}{x-4}}=$$

$$=e^{\lim\limits_{x\to\infty}\frac{6x}{x-4}}=\left\|\frac{\infty}{\infty}\right\|=e^{\lim\limits_{x\to\infty}\frac{6x/x}{\frac{x}{x}-\frac{4}{x}}}=$$

$$=e^{\lim\limits_{x\to\infty}\frac{6}{1-\frac{4}{x}}}=e^{\frac{6}{1-0}}=e^{6}$$

USE OCTAVE
https://octave-online.net/

octave:18> f=((x+2)/(x-4))^x
f = (sym)

$$\left(\frac{x + 2}{x - 4}\right)^{x}$$

octave:19> limit(f,inf)
ans = (sym)

$$e^{6}$$

Example 23.

$$\lim_{x\to 0}\frac{\arctan(5x)}{\arcsin(2x)}=\left[\begin{array}{c}\arctan(5x)\sim 5x\\ \arcsin(2x)\sim 2x\end{array}\right]=\lim_{x\to 0}\frac{5x}{2x}=\frac{5}{2}$$

USE OCTAVE
https://octave-online.net/

octave:20> f=atan(5*x)/asin(2*x)
f = (sym)

$$\frac{\operatorname{atan}(5\cdot x)}{\operatorname{asin}(2\cdot x)}$$

octave:21> limit(f,0)
ans = (sym) 5/2

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Example 24.

$$\lim_{x\to 0}\frac{\cos(x)-\cos(3x)}{x^2}=\left[\frac{1-1}{0}\right]=\lim_{x\to 0}\frac{2\sin(2x)\cdot\sin(x)}{x^2}=$$

$$=\lim_{x\to 0}\frac{2\sin(2x)\cdot\sin(x)}{\frac{1}{2}\cdot 2x\cdot x}=4$$

Use formulas

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

USE OCTAVE

https://octave-online.net/

octave:22> f=(cos(x)-cos(3*x))/x^2

f = (sym)

```
cos(x) - cos(3·x)
─────────────────
        2
       x
```

octave:23> limit(f,0)

ans = (sym) 4

Example 25.

$$\lim_{x\to 0}\frac{1-\cos 4x}{5x}=\frac{0}{0}=\lim_{x\to 0}\frac{2\sin^2 2x}{5x}=\frac{2}{5}\lim_{x\to 0}\frac{\sin^2 2x}{x}=$$

$$=\frac{2}{5}\lim_{x\to 0}\frac{\sin 2x\cdot\sin 2x}{\frac{1}{2}\cdot 2x}=\frac{2}{5}\lim_{x\to 0}\frac{\sin 2x}{\frac{1}{2}}=\frac{2}{5}\cdot 2\lim_{x\to 0}(\sin 2x)^{\to 0}=\frac{4}{5}\cdot 0=0$$

USE OCTAVE

https://octave-online.net/

octave:24> f=(1-cos(4*x))/(5*x)

f = (sym)

$$\frac{1-\cos(4\cdot x)}{5\cdot x}$$

octave:25> limit(f,0)

ans = (sym) 0

Example 26.

$$\lim_{x\to 0}\frac{\sin 7x-\sin 5x}{\sin x}=\lim_{x\to 0}\frac{2\sin\frac{7x-5x}{2}\cos\frac{7x+5x}{2}}{\sin x}=\lim_{x\to 0}\frac{2\sin x\cos 6x}{\sin x}=\lim_{x\to 0}(2\cos 6x)=$$

$$=\lim_{x\to 0}(2\cos 6x)=2\lim_{x\to 0}\cos 6x=$$

$$=2\cdot\cos(4\cdot 0)=2\cdot 1=2.$$

USE this formula: $\sin x-\sin y=2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$

USE OCTAVE

https://octave-online.net/

octave:26> f=(sin(7*x)-sin(5*x))/sin(x)

f = (sym)

$$\frac{-\sin(5\cdot x)+\sin(7\cdot x)}{\sin(x)}$$

octave:27> limit(f,0)

ans = (sym) 2