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Application of
the elements of
vector algebra to
Business and Economics

Vectors

Ordered n-tuple of objects is called a vector

$$\mathbf{y} = (y_1, y_2, \ldots, y_n).$$

Throughout the text we confine ourselves to vectors the elements y_i of which are real numbers.

In contrast, a variable the value of which is a single number, not a vector, is called *scalar*.

Example 2.1. We can describe some economic unit **EU** by the vector

EU= (output, # of employees, capital stock, profit)

Example 2.2. Let $\mathbf{EU}_1 = (Y_1, L_1, K_1, P_1)$ be a vector representing an economic unit, say, a firm, see Example 2.1 (where, as usually, Y is its output, L is the number of employees, K is the capital stock, and P is the profit). Let us assume that it is merged with another firm represented by a vector $\mathbf{EU}_2 = (Y_2, L_2, K_2, P_2)$ (that is, we should consider two separate units as a single one). The resulting unit will be represented by a sum of two vectors

$$\mathbf{EU}_3 = (Y_1 + Y_2, L_1 + L_2, K_1 + K_2, P_1 + P_2) = \mathbf{EU}_1 + \mathbf{EU}_2.$$

In this situation, we have also $\mathbf{EU}_2 = \mathbf{EU}_3 - \mathbf{EU}_1$. Moreover, if the second firm is similar to the first one, we can assume that $\mathbf{EU}_1 = \mathbf{EU}_2$, hence the unit

$$\mathbf{EU}_3 = (2Y_1, 2L_1, 2K_1, 2P_1) = 2 \cdot \mathbf{EU}_1$$

gives also an example of the multiplication by a number 2.

This example, as well as other 'economic' examples in this book has an illustrative nature. Notice, however, that the profit of the merged firm might be higher or lower than the sum of two profits $P_1 + P_2$.

Dot Product of Two Vectors

Definition 2.1. For any two vectors $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$, the *dot product*[4] of $\mathbf{x}$ and $\mathbf{y}$ is denoted by $(\mathbf{x}, \mathbf{y})$, and is defined as

$$(\mathbf{x}, \mathbf{y}) = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n = \sum_{i=1}^{n} x_i y_i . \tag{2.2}$$

[4]Other terms for dot product are *scalar product* and *inner product*.

Example 2.5 (Household expenditures). Suppose the family consumes n goods. Let $\mathbf{p}$ be the vector of prices of these commodities (we assume competitive economy and take them as given), and $\mathbf{q}$ be the vector of the amounts of commodities consumed by this household. Then the total expenditure of the household can be obtained by dot product of these two vectors

$$\mathrm{E} = (\mathbf{p}, \mathbf{q}).$$

An Economic Example: Two Plants

Consider a firm operating two plants in two different locations. They both produce the same output (say, 10 units) using the same type of inputs. Although the amounts of inputs vary between the plants the output level is the same.

The firm management suspects that the production cost in Plant 2 is higher than in Plant 1. The following information was collected from the managers of these plants.

PLANT 1

Input	Price	Amount used
Input 1	3	9
Input 2	5	10
Input 3	7	8

PLANT 2

Input	Price	Amount used
Input 1	4	8
Input 2	7	12
Input 3	3	9

Question 1. Does this information confirm the suspicion of the firm management?

Answer. In order to answer this question one needs to calculate the cost function. Let w_{ij} denote the price of the ith input at the jth plant and x_{ij} denote the quantity of ith input used in production jth plant ($i = 1, 2, 3$ and $j = 1, 2$). Suppose both of these magnitudes are perfectly divisible, therefore can be represented by real numbers. The cost of production can be calculated by multiplying the amount of each input by its price and summing over all inputs.

This means price and quantity vectors (**p** and **q**) are defined on real space and inner product of these vectors are defined. In other words, both **p** and **q** are in the space $\mathbb{R}^3$. The cost function in this case can be written as an inner product of price and quantity vectors as

$$c = (\mathbf{w}, \mathbf{q}), \tag{2.8}$$

where c is the cost, a scalar. Using the data in the above tables cost of production can be calculated by using (2.8) as:

In Plant 1 the total cost is 133, which implies that unit cost is 13.3.

In Plant 2, on the other hand, cost of production is 143, which gives unit cost as 14.3 which is higher than the first plant.

That is, the suspicion is reasonable.

Question 2. The manager of the Plant 2 claims that the reason of the cost differences is the higher input prices in her region than in the other. Is the available information supports her claim?

Answer. Let the input price vectors for Plant 1 and 2 be denoted as $\mathbf{p}_1$ and $\mathbf{p}_2$. Suppose that the latter is a multiple λ of the former, i.e.,

$$\mathbf{p}_2 = \lambda \mathbf{p}_1.$$

Since both vectors are in the space $\mathbb{R}^3$, length is defined for both. From the definition of length one can obtain that

$$|\mathbf{p}_2| = \lambda |\mathbf{p}_1|.$$

In this case, however as can be seen from the tables this is not the case. Plant I enjoys lower prices for inputs 2 and 3, whereas Plant 2 enjoys lower price for input 3. For a rough guess, one can still compare the lengths of the input price vectors which are

$$|\mathbf{p}_1| = 9.11, |\mathbf{p}_2| = 8.60,$$

which indicates that price data does not support the claim of the manager of the Plant 2. When examined more closely, one can see that the Plant 2 uses the most expensive input (input 2) intensely. In contrast, Plant 2 managed to save from using the most expensive input (in this case input 3). Therefore, the manager needs to explain the reasons behind the choice mixture of inputs in her plant.