DEFINITE INTEGRAL (part 1)

Example 1.

$$\int_1^2 \left(x^2-3x+1\right)dx=\int_1^2 x^2dx-3\int_1^2 xdx+\int_1^2 dx=\frac{x^3}{3}\Big|_1^2-3\cdot\frac{x^2}{2}\Big|_1^2+$$

$$+x\Big|_1^2=\frac{8}{3}-\frac{1}{3}-\frac{3}{2}(4-1)+2-1=\frac{7}{3}-\frac{9}{2}+1=-\frac{7}{6}.$$

Example 2.

$$\int_{-3}^{-2}\frac{dx}{1-x^2}=\int_{-2}^{-3}\frac{dx}{x^2-1}=\frac{1}{2}ln\left|\frac{x-1}{x+1}\right|\Big|_{-2}^{-3}=$$

$$=\frac{1}{2}\left(ln\left|\frac{-3-1}{-3+1}\right|-ln\left|\frac{-2-1}{-2+1}\right|\right)=\frac{1}{2}(ln2-ln3)=\frac{1}{2}ln\frac{2}{3}.$$

Example 3. Calculate $\int_0^1\frac{dx}{x^2+4x+5}.$

We allocate the perfect square from the square trinomial in the denominator

$x^2+4x+5=x^2+2\cdot 2x+4+1=(x+2)^2+1.$

We take the new variable $x+2=t,\quad dx=dt.$ We change limits of integration: if $x=0,$ then $t=2;$ if $x=1,$ then $t=3.$

$$\int_0^1\frac{dx}{x^2+4x+5}=\int_0^1\frac{dx}{(x+2)^2+1}=\int_2^3\frac{dt}{t^2+1}=arctgt\Big|_2^3=$$

$$=arctg3-arctg2.$$

Example 4. Calculate $\int_0^1 xe^{-x}dx.$

We use integration by parts:

$u=x,\quad dv=e^{-x}dx,$ then $du=dx,\ v=\int e^{-x}dx=-e^{-x}.$

We obtain

$$\int_0^1 xe^{-x}dx=x\left(-e^{-x}\right)\Big|_0^1-\int_0^1\left(-e^{-x}\right)dx=-e^{-1}+\int_0^1 e^{-x}dx=$$

$$=-\frac{1}{e}-e^{-x}\Big|_0^1=-\frac{1}{e}-\left(e^{-1}-e^0\right)=-\frac{1}{e}-\frac{1}{e}+1=\frac{e-2}{e}.$$

TASKS

Calculate definite integrals:

1. $\int_1^3 6x^2dx;$ **2.** $\int_1^5 \frac{7}{x}dx;$ **3.** $\int_1^3 e^{2x}dx;$ **4.** $\int_1^4 \sqrt{x}dx;$ **5.** $\int_9^{16} \frac{dx}{\sqrt{x}};$ **6.** $\int_1^e \frac{2}{x}dx$

7. $\int_0^1 (2x-1)^6 dx;$ **8.** $\int_0^8 \sqrt{2x}dx;$ **9.** $\int_0^8 \sqrt[3]{x}dx;$ **10.** $\int_{-1}^2 \left(x^2+2x+1\right)dx$

11. $\int_1^4 2xdx;$ **12.** $\int_3^8 \frac{dx}{x^2-6x+34};$ **13.** $\int_1^2 \left(\frac{4}{x}-5x^4+2\sqrt{x}\right)dx;$ **14.** $\int_1^8 \frac{dx}{\sqrt[3]{x^2}};$

15. $\int_0^{\pi/2} \cos x dx;$ **16.** $\int_0^{\pi/4} \sin 2x dx;$ **17.** $\int_{\pi/12}^{\pi/9} \cos 3x dx;$ **18.** $\int_0^{\pi/2} \cos^2 x dx;$

19. $\int_0^{5/2} \left(5x-2x^2\right)dx;$ **20.** $\int_{-4}^{-1/2} \frac{4x^3+2}{x^2}dx;$ **21.** $\int_0^1 \frac{2x}{1+x^2}dx;$ **22.** $\int_1^{\sqrt{2}} \frac{x}{\sqrt{4-x^2}}dx;$

23. $\int_1^2 \frac{3x^4-5x^2+7}{x}dx.$

Answer: $\frac{15}{4}+7\ln 2.$

24. $\int_4^9 \left(\frac{2x}{5}+\frac{1}{2\sqrt{x}}\right)dx.$

Answer: 14.

25. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}.$

Answer: $1-\frac{\sqrt{3}}{3}.$

26. $\int_2^3 \frac{xdx}{x^2+1}$

Answer: $\frac{1}{2}\ln 2.$

27. $\int_0^{\pi} \sin\frac{x}{2}\cos\frac{3x}{2}dx$

Answer: –1.

28. $\int_{-1}^0 \frac{dx}{x^2+2x+2}$

Answer: $\frac{\pi}{4}.$

29. $\int_0^{\frac{\pi}{2}} \sin^2 2x dx.$

Answer: $\frac{\pi}{4}.$

30. $\int_0^{\sqrt{3}-1} \frac{dx}{\sqrt{3-2x-x^2}}$

Answer: $\frac{\pi}{6}.$

31. $\int_0^1 x^4 dx;$

32. $\int_{-1/3}^0 \frac{dx}{\sqrt{2-6x-9x^2}};$ **33.** $\int_0^{-3} \frac{dx}{\sqrt{25+3x}};$ **34.** $\int_{-1/2}^1 \frac{dx}{\sqrt{8+2x-x^2}};$

35. $\int_1^2 \left(2x^2-4x+1\right)dx$; **36.** $\int_3^4 (x+1)dx$; **37.** $\int_{-2}^{-1} \frac{dx}{(11+5x)^3}$; **38.** $\int_3^4 \frac{dx}{25-x^2}$;

39. $\int_{-1}^{1} \left(6x^2-2x+5\right)dx$; **40.** $\int_1^8 \left(4x-\frac{1}{3\sqrt[3]{x^2}}\right)dx$; **41.** $\int_1^4 \frac{dx}{(1+2x)^2}$;

42. $\int_{-3}^{5} \sqrt[3]{5x+2}$; **43.** $\int_0^1 \sqrt{x+1}$; **44.** $\int_{\pi/3}^{\pi/2} \frac{dx}{\cos^2 \frac{x}{2}}$; **45.** $\int_4^{4\sqrt{3}} \frac{dx}{\sqrt{64-x^2}}$;

46. $\int_1^2 \left(x^5+\frac{1}{\sqrt{x}}\right)dx$ **47.** $\int_0^2 \left(x^3-x^2\right)dx$ **48.** $\int_0^1 x^2 dx$ **49.** $\int_0^1 (3x-1)^4 dx$

50. $\int_{-2}^{2} \left(x^5+5x^4-3x^2+x\right)dx$ **51.** $\int_0^1 2^x dx$ **52.** $\int_0^1 10^x dx$ **53.** $\int_1^3 x^2 dx$

54. $\int_1^3 \frac{dx}{x^2-4}$ **55.** $\int_1^2 5xdx$ **56.** $\int_0^1 \left(4x^3+1\right)dx$ **57.** $\int_{-3}^{1} \left(2x^2+3x-1\right)dx$

58. $\int_1^4 \left(2x+\frac{3}{\sqrt{x}}\right)dx$ **59.** $\int_{-2}^{4} \left(8+2x-x^2\right)dx$ **60.** $\int_1^2 \frac{dx}{x}$ **61.** $\int_1^9 \frac{dx}{\sqrt{x}}$ **62.** $\int_6^0 (1-x)dx$

Integration by substitution

Integration by parts

63. $\int_0^{\sqrt{5}} x\sqrt{x^2+4}dx$. *Answer*: $\frac{19}{3}$.

64. $\int_0^{\frac{\pi}{4}} \frac{dx}{1+\sin^2 x}$. *Answer*: $\frac{\sqrt{2}}{2} arctg\sqrt{2}$.

65. $\int_0^{\pi} x\sin x dx$. *Answer*: π.

66. $\int_1^e x\ln x dx$. *Answer*: $\frac{e^2+1}{4}$.

67. $\int_0^{\sqrt{3}} arctgx dx$. *Answer*: $\frac{\pi}{\sqrt{3}}-\ln 2$.

68. $\int_{-1}^{1} x^2 e^{-x} dx$. *Answer*: $\frac{e^2-5}{e}$.