TASKS

Task 1. Find $f(1;2)$, if $f(x,y)=\frac{2x-y}{x-2y}$.

Answer: 0.

Task 2. Find the domain of the definition of the function:

(1) $z=ln(x+y)$.

Answer: $(x+y>0)$

(2) $z=\frac{1}{x^2+y^2}$.

Answer: plane xOy, except the origin.

(3) $z=\frac{1}{x-1}+\frac{1}{y}$.

Answer: plane xOy, except points of straight lines $x=1$ and $y=0$

(4) $z=arcsin(x+y)$.

Answer: $x+y\le 1$ and $x+y\ge -1$. The domain between parallel straight lines $x+y=1$ and $x+y=-1$.

(5) $z=\sqrt{x-\sqrt{y}}$.

Answer: The domain which lies between the positive direction of x-axis and the parabola $y=x^2$ (including the boundary): $x\ge 0; y\ge 0; x^2\ge y$.

(6) $z=arctg\frac{x-y}{1+x^2y^2}$.

Answer: plane xOy.

(7) $z=\frac{\sqrt{4x-y^2}}{ln(1-x^2-y^2)}$.

Answer: The plane which lies inside the parabola $y^2=4x$, between the parabola and the circle $x^2+y^2=1$, including the arc of the parabola, except vertices and the arc of the circle.

(8) $z=\sqrt{\frac{x^2+2x+y^2}{x^2-2x+y^2}}$.

Answer: The domain which lies outside circles with centers $(-1;0)$ and $(1;0)$ and radiuses 1. Points of the first circle belong to the domain, points of the second circle don’t belong to the domain

(9) $z=\sqrt{y\sin x}$.

Answer: the domain

$2n\pi\le x\le(2n+1)\pi, y\ge 0$ and

$(2n+1)\pi\le x\le(2n+2)\pi, y\le 0,$ n

is a number.

Task 3. Find partial derivatives of the first order:

(1) $z = x^3 \cos y$.

(2) $z = tg\frac{x}{y}$ at the point $M(\pi, 1)$

(3) Prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z$, if $z = xy + xe^{\frac{y}{x}}$.

(4) $z = e^{x^2+y^2}$.

(5) $z = x^2 \sin^2 y$.

(6) $z = arctgxy$.

(7) $z = \arcsin(x + y)$.

(8) $z = e^{\sin\frac{y}{x}}$.

(9) $z = \ln\left(x + \sqrt{x^2 + y^2}\right)$.

(10) $z = \frac{1}{x^2 + y^2}$ at the point $(3;3)$

(11) Prove that the function $z = y\ln\left(x^2 - y^2\right)$ satisfies this equality $\frac{1}{x}\cdot\frac{\partial z}{\partial x} + \frac{1}{y}\cdot\frac{\partial z}{\partial y} = \frac{z}{y^2}$.

(12) Prove that the function $z = \frac{x^2}{2y} + \frac{x}{2} + \frac{1}{x} - \frac{1}{y}$ satisfies this equality $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = \frac{x^3}{y}$.

Task 4. Find the extremum of the function $z = f(x, y)$.

(1) $z = x^2 + xy + y^2 - 3x - 6y$.

Answer: $z_{min} = z(0;3) = -9$.

(2) $z = xy^2(1 - x - y)$.

Answer: $z_{max} = z\left(\frac{1}{4};\frac{1}{2}\right) = \frac{1}{64}$.

(3) $z = 3x^2 - y^2 + xy - 5x - 3y + 2$

Answer: $(1;-1)$, no extremums.

(4) $z = 2x^2 + y^2 - 6xy - 2x + 10y - 1$

Answer: $z_{\min} = z(2;1)$

(5) $z = x^3 - 7x^2 + xy - y^2 + 9x + 3y + 12$

Answer: $z_{max} = z(1;2) = 19$..

(6) $z = e^{-\left(x^2+y^2\right)}$

Answer: $z_{max} = z(0;0) = 1$

(7) $z = \frac{8}{x} + \frac{x}{y} + y, (x > 0, y > 0)$.

Answer: $z_{min} = z(4;2) = 6$

(8) $z = x^3 + 3xy^2 - 15x - 12y$.

Answer: $z_{min} = z(2;1) = -28$,

$z_{max} = z(-2;-1) = 28$

(9) $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

Answer: $z_{min} = z(0;0) = 0$