Vectors and operations with them.

A scalar, a cross and a mixed products of vectors

Tasks

1. Find coordinates of the vector $\vec{a} = \overrightarrow{AB} + \overrightarrow{CD}$ on the coordinate axes if $A(2,3,1)$, $B(4,1,-2)$, $C(6,3,7)$, $D(-5,-4,2)$ are given.

2. Find the scalar product of vectors $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{b} = -3\vec{i} + 4\vec{k}$ and the angle between them.

3. Calculate the area of the triangle ABC if $A(0,1,2)$, $B(-1,-3,5)$ and $C(1,4,-3)$ are given. Find the length of the altitude put down from the apex B.

4. Calculate the area of the parallelogram constructed on vectors $\vec{a} + 2\vec{b}$ and $2\vec{a} + \vec{b}$ if $|\vec{a}| = 1$, $|\vec{b}| = 2$, $\varphi = \frac{\pi}{6}$.

5. Show that the vectors $\vec{a} = 2\vec{i} + 5\vec{j} + 7\vec{k}$, $\vec{b} = \vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} + 2\vec{j} + 2\vec{k}$ are complanar.

6. Calculate the volume of the parallelepiped constructed on the vectors $\vec{a} = \vec{i} + \vec{j}$, $\vec{b} = \vec{j} + \vec{k}$ and $\vec{c} = \vec{i} + \vec{k}$.

7. At what value of m vectors $\vec{a} = m\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 4\vec{i} + m\vec{j} - 7\vec{k}$ are perpendicular?

8. At what values of α and β vectors $\vec{a} = \alpha\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 4\vec{i} + \beta\vec{j} - 7\vec{k}$ are parallel?

9. Find $\left(2\vec{a} + 4\vec{b}\right)\cdot\left(2\vec{a} - \vec{b}\right)$ if $|\vec{a}| = 3$, $|\vec{b}| = 2$, $\vec{a} \perp \vec{b}$.

10. Calculate the volume of the pyramid with the vertexes $A(1,0,0)$, $B(0,1,2)$, $C(0,0,5)$, $D(-4,2,2)$.

11. Find the cross product of the vectors $\vec{a} = -\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} + 2\vec{k}$.

12. Vertexes of the quadrangle are given: $A(1,-2,2)$, $B(1,4,0)$, $C(-4,1,1)$, $D(-5,-5,3)$. Show that its diagonals are perpendicular.

13. Find such a value α for which the vectors $\vec{a} = \alpha\vec{i} - 7\vec{j} + 5\vec{k}$ and $\vec{b} = 3\vec{i} + \alpha\vec{j} + 4\vec{k}$ are a) mutually perpendicular; b) parallel.

14. Two vectors $\vec{a} = -\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} + 2\vec{k}$ are given. Determine coordinates of the following vectors: 1) $\vec{a} + \vec{b}$; 2) $\vec{a} - \vec{b}$; 3) $-4\vec{a}$; 4) $-3\vec{a} + 2\vec{b}$.

15. Let the following vertexes of a pyramid be given: $A(1,-2,2)$, $B(4,1,-2)$, $C(-4,1,1)$, $D(-5,-4,2)$. Calculate the pyramid volume and the length of the altitude put down from the vertex D.

16. The vectors $\vec{a}$ and $\vec{b}$ form the angle of 45°. Find the area of the parallelogram constructed on the vectors $\vec{m} = \vec{a} - 2\vec{b}$ and $\vec{n} = 3\vec{a} + 2\vec{b}$ if $|\vec{a}| = 5$, $|\vec{b}| = 10$.