Laboratory work

Operations with matrices and determinants in MatLab/Octave/Octave / OCTAVE

The main purpose is

- forming students' basic mathematical knowledge for carrying out operations with matrices and determinants, calculation of the simplest determinants of matrices of the second and the third orders with the help of basic methods and properties, carrying out the simplest mathematical calculations (an addition, a subtraction, a multiplication);
- forming skills to use the mathematical instrument for carrying out operations with matrices and determinants with the help of MatLab/Octave.

Tasks:

1. Define matrices A and B of the size 3×3.
2. Define vector W as a matrix-column.
3. Calculate the matrix $A + B$; $A - B$; $-3A$.
4. Calculate the matrix $C = AB - BA$.
5. Calculate the matrix A^2 and a matrix of square elements of A. Compare results.
6. Check the equality $(A \cdot B)^T = B^T \cdot A^T$.
7. Calculate vector-column V as a product of the vector W and the matrix C.
8. Calculate the vector $Q = 5V - 3W$.
9. Calculate the determinant of the matrix C.
10. Exchange places of the first and the second columns of the matrix C. Calculate the determinant of the obtained matrix C_1. Compare determinants of matrices C and C_1.
11. Multiply elements of the second row of the matrix C by 2. Calculate the determinant of the obtained matrix C_2. Compare determinants of matrices C and C_2.
12. Compare determinants of matrices C and C^T.

13. Check whether the inverse matrix of the matrix A exists. If it exists, find it and display with its elements of the inverse matrix in the form of ordinary fractions.

14. Prove that the obtained result is correct.

Theoretical material MatLab/Octave/Octave software for carrying out operations with matrices

There are a lot of functions and commands for solving different problems of linear algebra.

Some basic functions and commands which are used for solving problems of matrix algebra are given in table 1.

Table 1

MatLab/Octave/Octave functions which are used for solving problems of matrix algebra

Function	Description
1	2

`A=[a11 a12 … a1n;…; am1 am2 … amn]`	Matrix size is $m \times n$. Rows are separated with the help of semicolon and elements of each row are separated with the help of semicolon space bar. For example,
	`>>A=[1 -3 2; 4 3 8; -2 6 1]`
	`A =`
	`1 -3 2`
	`4 3 8`
	`-2 6 1`

	`>>B=[-1;2;6]`
	`B =`
	`-1`
	`2`
	`6`

`A'`(apostrophe)	Transposition of a matrix which is defined as A. For example, a column can be defined as a transposed row:
	`>>B=[-1 2 6]`
	`B =`
	`-1 2 6`
	`>>B=B’`
	`B =`
	`-1`
	`2`

	6

A1+A2	Sum of two matrices, which are defined as variables A1 and A2 (matrices can have the same size). For example,
	`>>A1=[3 4;1 -2;0 -7]`
	`A1 =`
	`3 4`
	`1 -2`
	`0 -7`
	`>>A2=[2 -5;-9 -6; 8 5]`
	`A2 =`
	`2 -5`
	`-9 -6`
	`8 5`
	`>>A1+A2 % matrix sum`
	`ans =`
	`5 -1`
	`-8 -8`
	`8 -2`

a*A1±b*A2	Linear combination of matrices, which are defined as variables A1 and A2 (matrices can have the same size), a and b are numbers. For example,
	`>>3*A1-2*A2`
	`ans =`
	`5 22`
	`21 6`
	`-16 -31`

f(A)	If f(x) is an elementary function, then this operation leads to a calculation of a matrix of the same size with matrix A, each element of which equals a value of an elementary function of the corresponding element of matrix A. For example,
	`>>sin(A1)`
	`ans =`
	`0.1411 -0.7568`
	`0.8415 -0.9093`
	`0 -0.6570`
	`>>exp(B)`
	`ans =`
	`0.3679`
	`7.3891`
	`403.4288`

A.*C	Elementwise product of matrices, which are defined as matrices A and C. The result is a matrix of the same size with matrices A and C. For example,

	>>C=[2 1 0;3 1 3;4 -2 -7]; >>A.*C ans = 2 -3 0 12 3 24 -8 -12 -7 Matrices must have the same size, otherwise there will be a mistake. For example, >>A.*B ??? Error using ==> times Matrix dimensions must agree.
A./C	Elementwise division of matrices, which are defined as matrices A and C. The result is a matrix of the same size with matrices A and C. For example, >>C=[2 1 0;3 1 3;4 -2 -7]; >>A./C Warning: Divide by zero. ans = 0.5000 -3.0000 Inf 1.3333 3.0000 2.6667 -0.5000 -3.0000 0.1429 Matrices must have the same size.
A.^a	Each element of a matrix, which is defined as A raises to the power a. >>A.^2%raising to the power 2 ans = 1 9 4 16 9 64 4 36 1 This operation cab be used any rectangular matrix: >>A1.^2 % A1 is a rectangular matrix ans = 9 16 1 4 0 49
A*C	A product of matrices A and C. Sizes must be connected (the number of columns of A must be equal to the number of rows of C). For example: >>A*C ans = 1 -6 -23 49 -9 -47 18 2 11 If sizes are not connected, otherwise there will be a mistake: >>A1*A2

	??? Error using ==> mtimes Inner matrix dimensions must agree. >>A1*A2’ %sizes are connected, the result %is a product of matrix A1 and %the transposed matrix A2 ans = -14 -51 44 12 3 -2 35 42 -35
A^n	Raising a matrix to a power n (a matrix must be square). >>A^2 %raising to the power 2
	ans = -15 0 -20 0 45 40 20 30 45
size(A)	Defines a size of a matrix A. For example: >>size(A) ans = 3 3 >>size(B) ans = 3 1
A(i,j)	Refers to an element of matrix A, which is located in row i and column j. For example: >>A(2,3) ans = 8 >>B(1) ans = -1
A(i,:),A(:,j)	Refers to elements of matrix A, which is located in row i and column j. For example: >>A(2,:)%row 2 ans = 4 3 8 >>A(:,1)%column 1 ans = 1 4 -2

`det(A)`	Computes determinant of a square matrix A. For example:
	`>>det(A)`
	`ans =`
	`    75`

`zeros(m,n)`	Creates an array of zeros with the size $m \times n$. For example:
	`>>zeros(2,3)`
	`ans =`
	`     0     0     0`
	`     0     0     0`

`ones(m,n)`	Creates an array of ones with the size $m \times n$. For example:
	`>>ones(2,3)`
	`ans =`
	`     1     1     1`
	`     1     1     1`

`diag(B)`	Creates a diagonal square matrix B. For example:
	`>>diag(B)`
	`ans =`
	`    -1     0     0`
	`     0     2     0`
	`     0     0     6`

`eye(n)`	Creates an identity matrix of size $n \times n$. For example
	`>>eye(3) %`
	`ans =`
	`     1     0     0`
	`     0     1     0`
	`     0     0     1`

`inv(A)`	Computes an inverse matrix of a matrix A (a determinant is not equal to zero). For example:
	`>>inv(A)`
	`ans =`
	`   -0.6000    0.2000    0.4000`
	`   -0.2667    0.0667         0`
	`    0.4000         0    0.2000`

`rank(A)`	Computes rank of a matrix

prod(x)	Calculates a product of vector elements. For example: >>x=[1 3 -3 2 -5 1 8] x = 1 3 -3 2 -5 1 8 >>prod(x) ans = 720
sum(x)	Calculates a sum of vector elements. For example: >>sum(x) ans = 7
min(x)	Finds a minimal element of vector elements. For example: >>min(x) ans = -5
max(x)	Finds a maximal element of vector elements. For example: >>max(x) ans = 8
sort(x)	Sorts elements of a vector by increasing: >>sort(x) ans = -5 -3 1 1 2 3 8 Sorts elements of a vector by decreasing: >>-sort(-x) ans = 8 3 2 1 1 -3 -5
mean(x)	Знаходження середнього значення елементів вектора: >>mean(x) ans = 1

Theoretical material MatLab/Octave/Octave software for matrices and determinants

Let's demonstrate MatLab/Octave/Octave functions and operations for carrying out practical tasks for given matrices A, B and vector W :

$$\mathrm{A} = \begin{pmatrix} 3 & 1 & 0 \\ 6 & 13 & 1 \\ 1 & 2 & 1 \end{pmatrix}; \qquad \mathrm{B} = \begin{pmatrix} 4 & 5 & 6 \\ 1 & 0 & -1 \\ -1 & -1 & 2 \end{pmatrix}; \quad \mathrm{W} = \begin{pmatrix} 5 & -8 & 0 \end{pmatrix}$$

1. Define matrices A and B with size 3×3 in MatLab/Octave.

```
>> A=[3 1 0;6 13 1;1 2 1];%defines matrix A
>> B=[4 5 6;1 0 -1;-1 -1 2];% defines matrix B
>> A, B
A =
     3     1     0
     6    13     1
     1     2     1
B =
     4     5     6
     1     0    -1
     1    -1     2
```

2. Define vector W as vector-column in MatLab/Octave.

```
>> W=[5;-8;0]%defines vector-column W
W =
5
8
0
```

3. Calculate $A+B$; $A-B$; $-3A$.

```
>> A+B % calculates A+B, the result displays
ans =
     7     6     6
     7    13     0
     0     1     3
>> A-B % calculates A-B, the result displays
ans =
     1    -4    -6
     5    13     2
     2     3    -1
>> -3*A % calculates -3A, the result displays
ans =
     9    -3     0
    18   -39    -3
     3    -6    -3
```

4. Calculate matrix $C=AB-BA$.

```
>> C=A*B-B*A % calculates C=AB-BA, the result displays
C =
    35   -66     6
    34    30    26
```

```
    12    14     5
```

Thus, this operation $AB \neq BA$ is not fulfilled..

5. Calculate the matrix A^2 and a matrix of square elements of A. Compare results.

```
>> A*A %calculates a square of A
ans =
    15    16     1
    97   177    14
    16    29     3
>>A.^2 % calculates a matrix of square elements of A
ans =
     9     1     0
    36   169     1
     1     4     1
```

Thus, we have two different matrices where A^2 is A multiplied by itself, but it is not each element to the power two.

6. Check the inequality $(A \cdot B)^T = B^T \cdot A^T$.

```
>> (A*B)' %a calculation of a product of two matrices and its
          %transposition
ans =
    13    36     5
    15    29     4
    17    25     6
>> B'*A' % calculation of a product of transposed matrices
ans =
    13    36     5
    15    29     4
    17    25     6
```

Both matrices coincide, then this equality is fulfilled.

7. Calculate the vector-column V, which equals the product of the vector W and the matrix C.

```
>> V=C*W %calculation of a vector V
V =
353
70
52
```

8. Calculate the vector $Q = 5V - 3W$.

```
>> Q=5*V-3*W
```

```
Q =
  1750
   326
   260
```

Indeed, the element $q_1 = 5 \cdot v_1 - 3 \cdot w_1 = 5 \cdot 353 - 3 \cdot 5 = 5 \cdot 350 = 1750$, and similar equalities are carried out for the second and the third elements.

9. Calculate the determinant of the matrix C.

```
>> det(C)
ans =
   -1186
```

10. Exchange places of the first and the second columns of the matrix C. Calculate the determinant of the obtained matrix C_1. Compare determinants of matrices C and C_1.

```
>> C1=C;C1(:,1)=C(:,2);C1(:,2)=C(:,1) % change places of columns
C1 =
    66   -35     6
    30    34    26
    14    12     5

>> C % displays matrix C
C =
    35   -66     6
    34    30    26
    12    14     5

>> det(C1) % calculation of the determinant of matrix C1
ans =
    1186
```

At transposition of neighboring columns a determinant changes a sign знак on opposite.

11. Multiply elements of the second row of the matrix C by 2. Calculate the determinant of the obtained matrix C_2. Compare determinants of matrices C and C_2.

```
>> C2=C;C2(2,:)=C2(2,:).*2 % multiply elements of
                           %the second row by 2
C2 =
```

```
      35   -66     6
      68    60    52
      12    14     5
>> det(C2)
ans =
    2372
>> 2*det(C)
ans =
    2372
```

Let’s note that the property of determinants is carried out: if multiply elements of any row by any non-zero number, then a determinant will be multiplied by this number.

In this case the multiplication of elements of the second row by two was lead to increasing this determinant twice.

12. Compare determinants of matrices C and C^T.

```
>> det(C)% determinant C
ans =
    1186
>> det(C’)% determinant of the transposed matrix
ans =
    1186
```

Let’s note that the property of determinants is carried out: the determinant of the matrix C equals the determinant of the matrix C^T.

13. Check whether the inverse matrix of the matrix A exists. If it exists, find it and display with its elements of the inverse matrix in the form of ordinary fractions.

```
>> det(A) % calculates the matrix determinant
ans =
    28
```

The determinant isn’t equal to zero, thus there exists the inverse matrix.

```
>> format rational %since we need to present the result
                   %in the form ordinary fractions, we change
                   %the format
```

```
>> Ao=inv(A)% calculates the inverse matrix Ao
Ao =
    11/28          -1/28          1/28
     5/28           3/28         -3/28
     1/28          -5/28         33/28
```

14. Prove that the obtained result is correct.

By the definition the inverse matrix, if we multiply the matrix by it (on the left and right), then we obtain the unit matrix.

```
>> Ao*A % multiplication on the left
ans =
       1              *              0
       *              1              0
       *              0              1
>> A*Ao %multiplication on the right
ans =
        1              0              0
        *              1              *
        *              0              1
```

The sign `*` means almost zero, let's demonstrate it, changing the format:

```
>> format short
>> Ao*A
ans =

   1.0000   -0.0000         0

   0.0000    1.0000         0

   0.0000         0    1.0000

>> A*Ao
ans =
    1.0000         0         0
   -0.0000    1.0000   -0.0000
    0.0000         0    1.0000
```

Thus, the inverse matrix was found correctly.