MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

Guidelines
for practical tasks of the educational discipline “Probability theory and mathematical statistics” for foreign and English-learning full-time students of the direction “Management”

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Харків. Вид. ХНЕУ, 2016

It was approved at the meeting of the department of higher mathematics and economic mathematical methods
Protocol № 1 from August 26, 2015.

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Guidelines for practical tasks of the educational discipline "Probability theory and mathematical statistics" for foreign and English-learning full-time students of the direction "Management" / compiled by Ie. Misiura. – Kh. : KNUE, 2016. – 36 p. (English, Ukrainian)

Methodical recommendations are intended for foreign and English-learning students of the direction "Management" for practical studies of "Random variables and their characteristics" of the discipline "Probability theory and mathematical statistics ".

The sufficient theoretical material and the number of solved typical examples of each theme give students the possibility to master "Random variables and their characteristics" and apply the obtained knowledge in practice on their own.

Practical tasks for self-work and the list of theoretical questions which promote improving and extending students' knowledge of all the themes are given.

It is recommended for foreign and English-learning full-time students of the direction "Management".

Подано методичні рекомендації для студентів-іноземців та студентів, що навчаються англійською мовою, напряму підготовки «Менеджмент» денної форми навчання для практичних занять з теми «Випадкові величини та їх характеристики» навчальної дисципліни «Теорія ймовірностей та математична статистика».

Викладено необхідний теоретичний матеріал та наведено типові приклади, які допоможуть найбільш повному засвоєнню матеріалу з теми «Випадкові величини та їх характеристики» та застосуванню отриманих знань на практиці.

Подано завдання для самостійної роботи та перелік теоретичних питань, що сприяють удосконаленню та поглибленню знань студентів з даної теми.

Рекомендовано для студентів-іноземців та студентів, що навчаються англійською мовою, напряму підготовки «Менеджмент» денної форми навчання.

Introduction

Methodical recommendations are intended for foreign and English-learning students of the direction "Management" for practical studies of “Random variables and their characteristics” of the discipline "Probability theory and mathematical statistics".

Its aim is the practical application of mathematic apparatus to the problem solutions in “Random variables and their characteristics”, which consists of the following themes: discrete random variables, their distribution laws and numerical characteristics; continuous random variables, a function and a density of a distribution of probabilities, numerical characteristics; a uniform, an exponential and a normal laws of probabilities distribution.

The sufficient number of solved typical examples of each theme gives students the possibility to master “Random variables and their characteristics” and apply the obtained knowledge in practice on their own. At the end of methodical recommendations there are tasks for self-work and the list of theoretical questions which promote improving and extending students’ know-ledge of all the themes.

2. Continuous random variables

A *continuous random variable* is a random variable where the data can take infinitely many values on some numerical interval or a random variable which takes an infinite number of possible values. Continuous random variables are usually measurements.

Examples include height, weight, the amount of sugar in an orange, the time required to run a mile.

A continuous random variable is characterized by two functions: a density function (the differential distribution function) $f(x)$ and a distribution function (the integral distribution function) $F(x)$.

The random variable X is called a *continuous random variable*, if for any numbers $a < b$ such non-negative function $f(x)$ exists, that

$$P(a < X < b) = \int_a^b f(x)dx.$$

The function $f(x)$ is called a *density function* of continuous random variable.

General properties of the density function:

1. $f(x)$ is a non-negative function, i.e. $f(x) \geq 0$ for all *x*.

2. $f(x)$ is a non-decreasing function for $x \in (-\infty, \infty)$, then

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

(the condition of normalization of the function $f(x)$).

3. The relationship between the functions $f(x)$ and $F(x)$:

$$F(x) = \int_{-\infty}^{x} f(x)dx.$$

The probability of the fact that a random variable X receives a value less than x, is called a *cumulative distribution function* of random variable X and marked as $F(x)$:

$$F(x) = P(X \leq x).$$

General properties of the integral distribution function:

1. $F(x)$ is a bounded function, i.e. $0 \leq F(x) \leq 1$.

2. $F(x)$ is a non-decreasing function for $x \in (-\infty, \infty)$, i.e. if $x_2 > x_1$, then $F(x_2) \geq F(x_1)$.

3. $\lim\limits_{x \to -\infty} F(x) = F(-\infty) = 0$.

4. $\lim\limits_{x \to +\infty} F(x) = F(+\infty) = 1$.

5. The relationship between the functions $f(x)$ and $F(x)$: $f(x) = F'(x)$.

6. The probability that a random variable X lies in the interval (x_1, x_2) is equal to the increment of its cumulative distribution function on this interval; i.e.

$$P(x_1 < X < x_2) = F(x_2) - F(x_1) = \int_{x_1}^{x_2} f(x)dx.$$

Example 2.1. The density function of a continuous variable X is given:

$$f(x) = \begin{cases} 0, & x \le 0 \\ c\left(4x - 2x^2\right), & 0 < x \le 2 \\ 0, & x > 2 \end{cases}.$$

a) What is the value of c? b) Find $P(X > 1)$.

Solution. Since $f(x)$ is a probability density function, we must have the condition of normalization of this function that $\int_{-\infty}^{\infty} f(x)dx = 1$, implying that

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0dx + \int_{0}^{2} c\left(4x - 2x^2\right)dx + \int_{2}^{+\infty} 0dx = 1$$

or

$$\int_{0}^{2} c\left(4x - 2x^2\right)dx = 1$$

or

$$c\left(4\frac{x^2}{2} - 2\frac{x^3}{3}\right)\Bigg|_0^2 = c\left(2x^2 - \frac{2x^3}{3}\right)\Bigg|_0^2 = 1$$

or

$$c\left(8 - \frac{16}{3} - 0\right) = c \cdot \frac{8}{3} = 1$$

or

$$c = \frac{3}{8}.$$

So, a probability density function is

$$f(x) = \begin{cases} 0, & x \le 0 \\ \frac{3}{8}\left(4x - 2x^2\right), & 0 < x \le 2 \\ 0, & x > 2 \end{cases}$$

Let's find $P(X > 1)$:

$$P(X>1)=P(1<X<+\infty)=\int_1^{+\infty} f(x)dx=\int_1^2 \frac{3}{8}\left(4x-2x^2\right)dx+\int_2^{+\infty} 0dx=$$

$$=\frac{3}{8}\left(2x^2-\frac{2x^3}{3}\right)\Bigg|_1^2=\frac{3}{8}\left(\left(2\cdot 2^2-\frac{2\cdot 2^3}{3}\right)-\left(2\cdot 1^2-\frac{2\cdot 1^3}{3}\right)\right)=\frac{3}{8}\left(\frac{8}{3}-\frac{4}{3}\right)=\frac{1}{2}.$$

Example 2.2. The differential distribution function of a continuous variable X is given:

$$f(x)=\begin{cases}0, & x\le 1\\ x-\dfrac{1}{2}, & 1<x\le 2.\\ 0, & x>2\end{cases}$$

Find the integral distribution function $F(x)$. Plot the graphs of the functions $f(x)$ and $F(x)$.

Solution. The integral distribution function $F(x)$ according to the formula is $F(x)=\int_{-\infty}^{x} f(x)dx$.

If $x\le 1$, then $f(x)=0$ and $F(x)=\int_{-\infty}^{x} f(x)dx=\int_{-\infty}^{x} 0dx=0$.

If $1<x\le 2$, then $F(x)=\int_{-\infty}^{x} f(x)dx=\int_{-\infty}^{1} f(x)dx+\int_1^{x} f(x)dx=$

$$=\int_{-\infty}^{1} 0dx+\int_1^{x}\left(x-\frac{1}{2}\right)dx=\left(\frac{x^2}{2}-\frac{1}{2}x\right)\Bigg|_1^{x}=\frac{x^2}{2}-\frac{1}{2}x-\left(\frac{1}{2}-\frac{1}{2}\right)=\frac{x^2}{2}-\frac{x}{2}.$$

If $x>2$, then $F(x)=\int_{-\infty}^{x} f(x)dx=\int_{-\infty}^{1} f(x)dx+\int_1^{2} f(x)dx+\int_2^{x} f(x)dx=$

$$=\int_{-\infty}^{1} 0dx+\int_1^{2}\left(x-\frac{1}{2}\right)dx+\int_2^{x} 0dx=\left(\frac{x^2}{2}-\frac{1}{2}x\right)\Bigg|_1^{2}=\frac{2^2}{2}-\frac{1}{2}\cdot 2-\left(\frac{1^2}{2}-\frac{1}{2}\cdot 1\right)=$$

$$=2-1-0=1.$$

Let’s write the formula for the integral distribution function $F(x)$:

$$F(x)=\begin{cases}0, & x\le 1\\ \dfrac{x^2}{2}-\dfrac{x}{2}, & 1<x\le 2\\ 1, & x>2\end{cases}.$$

Example 2.3. The integral distribution function of a continuous variable X is given:

$$F(X)=\begin{cases}0, & x\le 0{,}5\\ \dfrac{2x-1}{2}, & 0{,}5<x\le 1{,}5\\ 1, & x>1{,}5\end{cases}.$$

Find the density function $f(x)$.

Solution. It is known that $f(x)=F'(x)$. Thus

$$f(x)=F'(X)=\begin{cases}(0)', & x\le 0{,}5\\ \left(\dfrac{2x-1}{2}\right)', & 0{,}5<x\le 1{,}5\\ (1)', & x>1{,}5\end{cases}=\begin{cases}0, & x\le 0{,}5\\ \dfrac{2-0}{2}, & 0{,}5<x\le 1{,}5\\ 0, & x>1{,}5\end{cases}=$$

$$=\begin{cases}0, & x\le 0{,}5\\ 1, & 0{,}5<x\le 1{,}5\\ 0, & x>1{,}5\end{cases}.$$

2.1. Numerical characteristics of continuous random variables

1) *Mathematical expectation* $M(X)$ is

$$M(X)=\int_{-\infty}^{\infty}x\cdot f(x)dx.$$

2) *Variance* $D(X)$ is

$$D(X)=\int_{-\infty}^{\infty} x^2\cdot f(x)dx-[M(X)]^2 \text{ or } D(X)=\int_{-\infty}^{\infty}(x-M(X))^2\cdot f(x)dx.$$

3) *Root-mean-square deviation (or standard deviation)* $\sigma(X)$ is

$$\sigma(X)=\sqrt{D(X)}.$$

Example 2.4. The density function of a continuous variable X is given:

$$f(x)=\begin{cases}0, & x\le 0{,}5\\ 1, & 0{,}5<x\le 1{,}5\\ 0, & x>1{,}5\end{cases}.$$

Calculate: a) the mathematical expectation $M(X)$ and the variance $D(X)$; b) the probability that a random variable X lies in the interval $(1.0,\,2.3)$; c) plot the graphs of the functions $F(X)$ and $f(x)$.

Solution. a) Let's find the mathematical expectation:

$$M(X)=\int_{-\infty}^{\infty} x\cdot f(x)dx=\int_{-\infty}^{0.5} x\cdot 0dx+\int_{0.5}^{1.5} x\cdot 1dx+\int_{1.5}^{\infty} x\cdot 0dx=$$

$$=0+\int_{0.5}^{1.5} xdx+0=\left.\frac{x^2}{2}\right|_{0.5}^{1.5}=\frac{1}{2}\cdot\left[1.5^2-0.5^2\right]=\frac{1}{2}\cdot[2.25-0.25]=\frac{1}{2}\cdot 2=1.$$

Let's calculate the variance:

$$D(X)=\int_{-\infty}^{\infty} x^2\cdot f(x)dx-[M(X)]^2=$$

$$=\int_{-\infty}^{0.5} x^2\cdot 0dx+\int_{0.5}^{1.5} x^2\cdot 1dx+\int_{1.5}^{\infty} x^2\cdot 0dx-1^2=0+\int_{0.5}^{1.5} x^2dx+0-1=$$

$$=\left.\frac{x^3}{3}\right|_{0.5}^{1.5}-1=\frac{1}{3}\cdot\left[1.5^3-0.5^3\right]-1=\frac{1}{3}\cdot[3.375-0.125]-1=$$

$$=\frac{1}{3}\cdot 3.25-1\approx 0.0833.$$

b) Let's find the probability that a random variable X lies in the interval $(1.0,\ 2.3)$ by the formula $P(x_1 < X < x_2) = F(x_2) - F(x_1)$.

Thus,

$$P(1 < X < 2.3) = F(2.3) - F(1) = 1 - \frac{2 \cdot 1 - 1}{2} = 1 - \frac{1}{2} = \frac{1}{2} = 0.5 .$$

c) Let's plot the graphs of the functions $F(X)$ and $f(x)$ (Fig. 2.5, 2.6).

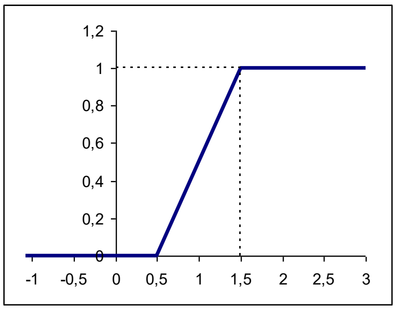

Fig. 2.5. **The graph of** $F(X)$

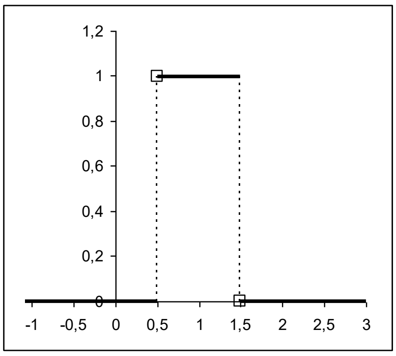

Fig. 2.6. **The graph of** $f(x)$

2.2. Main continuous distributions

2.2.1. Uniform distribution law. The uniform law of distribution is characterized by two functions: a probability density function $f(x)$ (the differential distribution function) (Fig. 2.1) and a cumulative distribution function $F(X)$ (the integral distribution function) (Fig. 2.2).

$$f(x) = \begin{cases} 0, & x \le a \\ \dfrac{1}{b-a}, & a < x \le b, \\ 0, & x > b \end{cases} \qquad F(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a < x \le b. \\ 1, & x > b \end{cases}$$

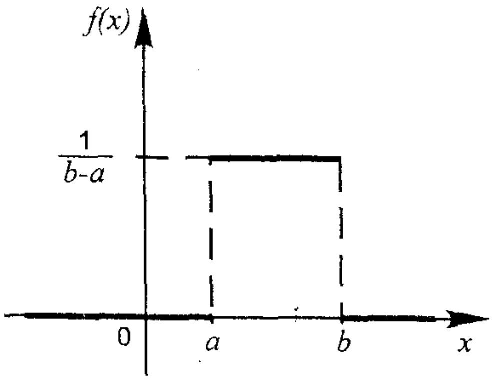

Fig. 2.1. **The graph of** $f(x)$

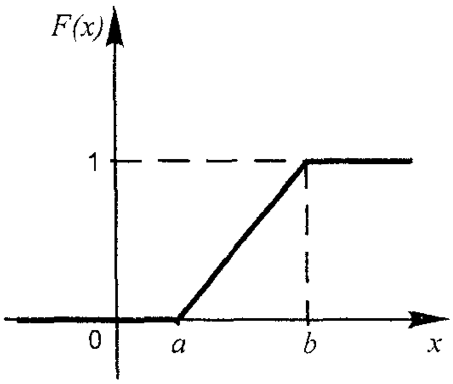

Fig. 2.2. **The graph of** $F(X)$

The probability that a random variable X lies in the interval (α, β) is equal to the increment of its integral distribution function on this interval; i.e.

$$P(\alpha < X < \beta) = F(\beta) - F(\alpha) = \frac{\beta - \alpha}{b - a}.$$

Numerical characteristics of uniform random variables are

1) *mathematical expectation* $M(X)$:

$$M(X) = \frac{a+b}{2};$$

2) *variance* $D(X)$:

$$D(X) = \frac{(b-a)^2}{12};$$

3) *root-mean-square deviation (or standard deviation)* $\sigma(X)$:

$$\sigma(X) = \frac{b-a}{2\sqrt{3}}.$$

Example 2.5. The parameters a, b of the uniform law of distribution are given: $a = 2$ and $b = 6$. Find: a) functions $f(x)$ and $F(x)$; b) the mathematical expectation $M(X)$, the variance $D(X)$ and the root-mean-square deviation $\sigma(X)$; c) $P(0 < X < 3)$.

Solution. Let's find functions $f(x)$ and $F(x)$ substituting $a = 2$ and $b = 6$ into formulas for functions:

$$f(x)=\begin{cases}0, & x\le 2\\ \dfrac{1}{6-2}, & 2<x\le 6\\ 0, & x>6\end{cases}=\begin{cases}0, & x\le 2\\ \dfrac{1}{4}, & 2<x\le 6,\\ 0, & x>6\end{cases}$$

$$F(x)=\begin{cases}0, & x\le 2\\ \dfrac{x-2}{6-2}, & 2<x\le 6\\ 1, & x>6\end{cases}=\begin{cases}0, & x\le 2\\ \dfrac{x-2}{4}, & 2<x\le 6.\\ 1, & x>6\end{cases}$$

Let's calculate the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$:

$$M(X)=\frac{a+b}{2}=\frac{2+6}{2}=4,\qquad D(X)=\frac{(6-2)^2}{12}=\frac{4^2}{12}=\frac{4}{3},$$

$$\sigma(X)=\frac{b-a}{2\sqrt{3}}=\frac{6-2}{2\sqrt{3}}=\frac{4}{2\sqrt{3}}=\frac{2}{\sqrt{3}}.$$

Let's calculate the probability that a uniform random variable X lies in the interval $(0,3)$:

$$P(\alpha<X<\beta)=P(0<X<3)=F(3)-F(0)=\frac{3-0}{6-2}=\frac{3}{4}=0.75.$$

2.2.2. Exponential distribution law. The exponential law of distribution is characterized by two functions: a probability density function $f(x)$ (the differential distribution function) (Fig. 2.3) and a cumulative distribution function $F(X)$ (the integral distribution function) (Fig. 2.4).

$$f(x)=\begin{cases}0, & x<0\\ \lambda\cdot e^{-\lambda x}, & x\ge 0\end{cases},\qquad F(x)=\begin{cases}0, & x<0\\ 1-e^{-\lambda x}, & x\ge 0\end{cases}.$$

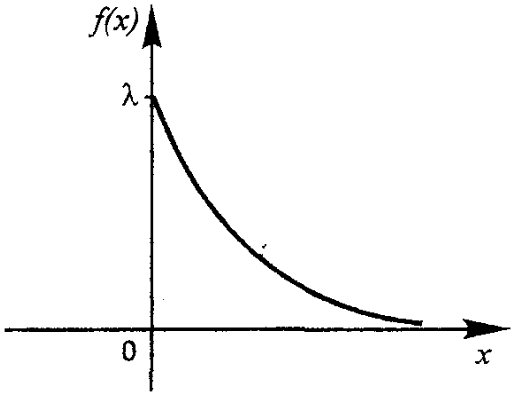

Fig. 2.3. **The graph of** $f(x)$

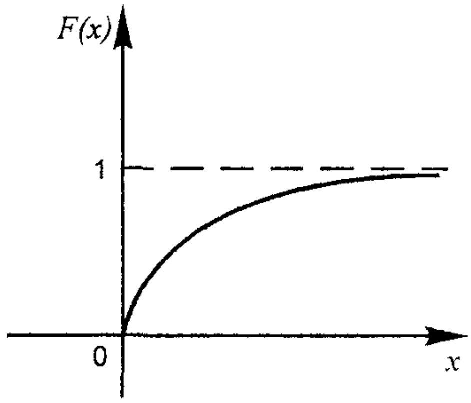

Fig. 2.4. **The graph of** $F(X)$

The probability that a random variable X lies in the interval (α, β) is equal to the increment of its integral distribution function on this interval; i.e.

$$P(\alpha < X < \beta) = F(\beta) - F(\alpha) = e^{-\lambda\alpha} - e^{-\lambda\beta}.$$

Numerical characteristics of exponential random variables are

1) *mathematical expectation* $M(X)$:

$$M(X) = \frac{1}{\lambda}.$$

2) *variance* $D(X)$:

$$D(X) = \frac{1}{\lambda^2},$$

3) *root-mean-square deviation (or standard deviation)* $\sigma(X)$:

$$\sigma(X) = \frac{1}{\lambda}.$$

Example 2.6. The probability density function of the exponential law of distribution $f(x) = \begin{cases} 0, & x < 0 \\ 0.05 \cdot e^{-0.05x}, & x \geq 0 \end{cases}$ is given. Find: a) the function $F(x)$; b) the mathematical expectation $M(X)$, the variance $D(X)$ and the root-mean-square deviation $\sigma(X)$; c) $P(2 < X < 10)$.

Solution. We have that $\lambda = 0.05$ from the formula of $f(x)$. Let's substitute this parameter into the formula for $F(x)$ and find the numerical characteristics:

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & x \geq 0 \end{cases} = \begin{cases} 0, & x < 0 \\ 1 - e^{-0.05x}, & x \geq 0 \end{cases},$$

$$M(X) = \frac{1}{\lambda} = \frac{1}{0.05} = 20,\ D(X) = \frac{1}{\lambda^2} = \frac{1}{0.05^2} = 400,\ \sigma(X) = \frac{1}{\lambda} = \frac{1}{0.05} = 20.$$

Let's calculate the probability $P(2 < X < 10)$ that a random variable X lies in the interval $(2,10)$:

$$P(\alpha < X < \beta) = P(2 < X < 10) = e^{-\lambda\alpha} - e^{-\lambda\beta} = e^{-0.05\cdot 2} - e^{-0.05\cdot 10} =$$
$$= 0.90484 - 0.60653 = 0.29831.$$

2.2.3. Normal law distribution. The normal law of distribution is characterized by two functions: a probability density function $f(X)$ (the differential distribution function) (Fig. 2.5) and a cumulative distribution function $F(X)$ (the integral distribution function) (Fig. 2.6)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}} = \frac{\varphi(t)}{\sigma}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-a)^2}{2\sigma^2}} dx = \frac{1}{2} + \Phi(t),$$

where $\varphi(t) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t^2}{2}}$ and $\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_0^t \cdot e^{-\frac{t^2}{2}} dt$ are Laplace differential and integral functions (see Table 1), $t = \frac{x-a}{\sigma}$ is a normalized value

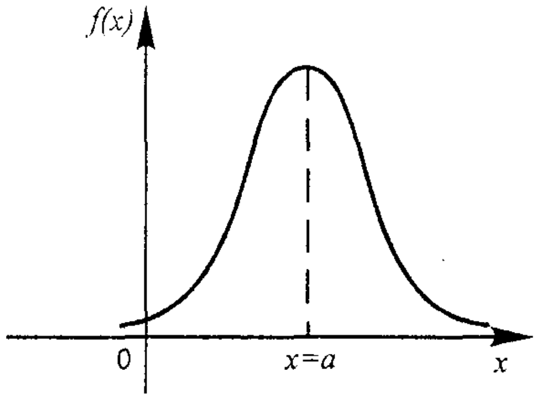

Fig. 2.5. **A density curve** $f(x)$ **graph**

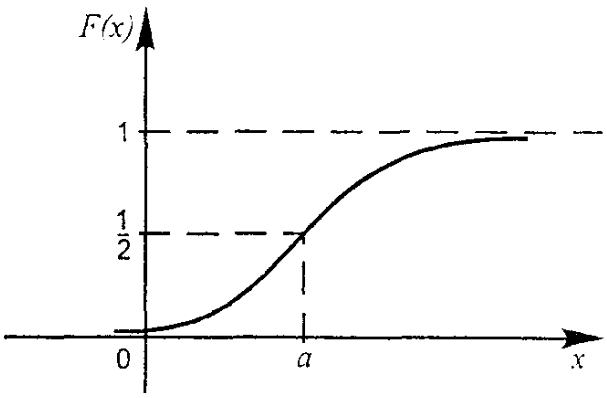

Fig. 2.6. **The graph of** $F(X)$

The probability that a random variable X lies in the interval (α, β) is equal to the increment of its integral distribution function on this interval; i.e.

$$P(\alpha < X < \beta) = F(\beta) - F(\alpha) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right).$$

Numerical characteristics of exponential random variables are

1) *mathematical expectation* $M(X)$:

$$M(X) = a \qquad (M(t) = 0),$$

2) *variance* $D(X)$:

$$D(X) = \sigma^2 \qquad (D(t) = 1),$$

3) *root-mean-square deviation (or standard deviation)* $\sigma(X)$:

$$\sigma(X) = \sigma \qquad (\sigma(t) = 1).$$

Example 2.7. The probability density function of the normal law of distribution $f(x) = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(x-3)^2}{8}}$ is given. Find the integral function, calculate the numerical characteristics and $P(1 < X < 7)$.

Solution. We have that $a = 3$ and $\sigma = 2$ from the formula of $f(x)$. Let's substitute these parameters into the formula for $F(x)$ and find the numerical characteristics:

$$F(x)=\int_{-\infty}^{x}\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-a)^2}{2\sigma^2}}dx=\int_{-\infty}^{x}\frac{1}{2\sqrt{2\pi}}\cdot e^{-\frac{(x-3)^2}{2\cdot 2^2}}dx=$$

$$=\int_{-\infty}^{x}\frac{1}{2\sqrt{2\pi}}\cdot e^{-\frac{(x-3)^2}{8}}dx,$$

$M(X)=a=3$, $D(X)=\sigma^2=2^2=4$ and $\sigma(X)=\sigma=2$.

Let's calculate the probability $P(1<X<7)$ that a random variable X lies in the interval $(1,7)$:

$$P(\alpha<X<\beta)=P(1<X<7)=\Phi\left(\frac{\beta-a}{\sigma}\right)-\Phi\left(\frac{\alpha-a}{\sigma}\right)=$$

$$=\Phi\left(\frac{7-3}{2}\right)-\Phi\left(\frac{1-3}{2}\right)=\Phi(2)-\Phi(-1)=\left\|\begin{matrix}\textit{use the property}\\ \Phi(-x)=\Phi(x)\end{matrix}\right\|=\Phi(2)+\Phi(1)=$$

$$=\left\|\begin{matrix}\textit{use table } 1\\ \Phi(2)=0.4772\\ \Phi(1)=0.3413\end{matrix}\right\|=0.4772-0.3413=0.1359.$$

Control tasks. Continuous random variables

2.1. The integral distribution function of a continuous variable X is given. Find: a) the density function $f(x)$; b) the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$; c) the probability $P(\alpha < X < \beta)$; d) plot the graphs of functions $F(X)$ and $f(x)$.

1) $$F(X)=\begin{cases}0, & x\le -2\\ \dfrac{x+2}{4}, & -2<x\le 2\\ 1, & x>2\end{cases}$$
$\alpha=0{,}5$, $\beta=4{,}5$;

2) $$F(X)=\begin{cases}0, & x\le 0{,}5\\ \dfrac{2x-1}{2}, & 0{,}5<x\le 1{,}5\\ 1, & x>1{,}5\end{cases}$$
$\alpha=1{,}0$, $\beta=2{,}3$;

3) $$F(X)=\begin{cases}0, & x\le 1\\ \dfrac{1}{4}(x-1)^2, & 1<x\le 3\\ 1, & x>3\end{cases}$$
$\alpha=0{,}2$, $\beta=2{,}6$;

4) $$F(X)=\begin{cases}0, & x\le 5\\ \dfrac{1}{4}(x-5)^2, & 5<x\le 7\\ 1, & x>7\end{cases}$$
$\alpha=3{,}4$, $\beta=6{,}6$.

2.2. The density function of a continuous variable X is given. Find: a) the density function $f(x)$; b) the numerical characteristics $M(X)$, $D(X)$, $\sigma(X)$; c) the probability $P(\alpha < X < \beta)$; d) plot graphs of $F(X)$ and $f(x)$.

1) $$f(x)=\begin{cases}0, & x\le 1\\ \dfrac{x}{4}, & 1<x\le 3\\ 0, & x>3\end{cases}$$
$\alpha=0$, $\beta=2$;

2) $$f(x)=\begin{cases}0, & x\le 1\\ \dfrac{1}{2}(x-1), & 1<x\le 3\\ 0, & x>3\end{cases}$$
$\alpha=1.5$, $\beta=5$;

3) $$f(x)=\begin{cases}0, & x\le 0,\\ \dfrac{3x^2}{8}, & 0<x\le 2,\\ 0, & x>2.\end{cases}$$
$\alpha=-1$, $\beta=1.5$;

4) $$f(x)=\begin{cases}0, & x\le 3,\\ \dfrac{1}{25}(x-3)^2, & 3<x\le 8,\\ 0, & x>8.\end{cases}$$
$\alpha=4$, $\beta=6$.

2.3. The density function of a continuous variable X is given:

1) $$f(x)=\begin{cases}0, & x\le 0\\ cx^2, & 0<x\le 1,\\ 0, & x>1\end{cases}$$

2) $$f(x)=\begin{cases}0, & x\le 2\\ c(x-2), & 2<x\le 5.\\ 0, & x>5\end{cases}$$

Find: a) the parameter c; b) the integral distribution function $F(x)$; c) the probability $P(0.5<X<3)$; d) the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$. Plot the graphs of the functions $f(x)$ and $F(x)$;

2.4. The density function of the exponential distribution $f(x)=\begin{cases}0, & x<0\\ 0.04\cdot e^{-0.04x}, & x\ge 0\end{cases}$ is given. Find: a) the function $F(x)$; b) $M(X)$, $D(X)$ and $\sigma(X)$; c) $P(1<X<2)$.

2.5. The waiting time, in hours, between successive speeders spotted by a radar unit is an exponential continuous random variable with cumulative distribution function $F(x)=\begin{cases}0, & x<0\\ 1-e^{-8x}, & x\ge 0\end{cases}$. Find: a) the density function $f(x)$; b) $M(X)$, $D(X)$ and $\sigma(X)$; c) the probability of waiting less than 12 minutes between successive speeders.

2.6. The mathematical expectation of a normal random variable X equals 20, the root-mean-square deviation equals 5. Find: a) functions $f(x)$ and $F(x)$; b) the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$; c) the probability that a random variable X lies in the interval $(15,25)$.

2.7. The mathematical expectation of a normal random variable X equals 25. It is known that $P(35<X<40)=0{,}2$. Find the probability $P(10<X<15)$.

2.8. The parameters a,b of the uniform law of distribution are given: $a=1$ and $b=4$. Find: a) the functions $f(x)$ and $F(x)$; b) $M(X)$, $D(X)$ and $\sigma(X)$; c) $P(0<X<3)$, $P(2<X<5)$.

2.9. A man's height is distributed by the normal law. The mathematical expectation of a normal random variable X equals 170 centimeters, the root-mean-square deviation equals 5 centimeters. Find the probability that the men' height there will be a) less than 160 centimeters; b) greater than 180 centimeters; c) from 160 to 175 centimeters.

2.10. The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has the density function $f(x)=\begin{cases} 0, & x \le 0 \\ \dfrac{2(x+2)}{4}, & 0 < x \le 1 \\ 0, & x > 1 \end{cases}$. Show that $P(0 < X < 1) = 1$. Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

2.11. Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function $f(x)=\begin{cases} 0, & x \le -1 \\ c\left(3-x^2\right), & -1 < x \le 1 \\ 0, & x > 1 \end{cases}$. Determine c that renders $f(x)$ a valid density function. Find the probability that a random error in measurement is less than 1/2. For this particular measurement, it is undesirable if the magnitude of the error, exceeds 0.8. What is the probability that this occurs?

Theoretical questions

1. Give a definition of a continuous random variable.
2. Give a definition of a cumulative distribution function and its properties.
3. Give a definition of a distribution density and its properties.
4. Write down the condition of normalization of a density function.
5. The numerical characteristics of a distribution (a mathematical expectation, a variance, a root-mean-square deviation) and their properties.
6. Write down the formula of the probability that a continuous random variable X lies in the interval (x_1, x_2).
7. A uniform distribution law and its numerical characteristics.
8. An exponential distribution law and its numerical characteristics.
9. A normal distribution law and its standard representation.
10. Classify the following random variables as discrete or continuous:

a) the number of automobile accidents per year in the city;

b) the length of time to play 18 holes of golf;

c) the amount of milk produced yearly by a particular cow;

d) the number of eggs laid each month by a hen;

e) the number of building permits issued each month in a certain city;

f) the weight of grain produced per acre.

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Appendix A

Table 1

Values of Laplace cumulative distribution function $\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{z^2}{2}} dz$

t	0	1	2	3	4	5	6	7	8	9
0,0	0,0000	0,0040	0,0080	0,0120	0,0160	0,0199	0,0239	0,0279	0,0319	0,0359
0,1	0,0398	0,0438	0,0478	0,0517	0,0557	0,0596	0,0636	0,0675	0,0714	0,0754
0,2	0,0793	0,0832	0,0871	0,0910	0,0948	0,0987	0,1026	0,1064	0,1103	0,1141
0,3	0,1179	0,1217	0,1255	0,1293	0,1331	0,1368	0,1406	0,1443	0,1480	0,1517
0,4	0,1554	0,1591	0,1628	0,1664	0,1700	0,1736	0,1772	0,1808	0,1844	0,1879
0,5	0,1915	0,1950	0,1985	0,2019	0,2054	0,2088	0,2123	0,2157	0,2190	0,2224
0,6	0,2258	0,2291	0,2324	0,2356	0,2389	0,2422	0,2454	0,2486	0,2518	0,2549
0,7	0,2580	0,2612	0,2642	0,2673	0,2704	0,2734	0,2764	0,2794	0,2823	0,2852
0,8	0,2881	0,2910	0,2939	0,2967	0,2996	0,3023	0,3051	0,3078	0,3106	0,3133
0,9	0,3159	0,3186	0,3212	0,3238	0,3264	0,3289	0,3315	0,3340	0,3365	0,3389
1,0	0,3413	0,3438	0,3461	0,3485	0,3508	0,3531	0,3554	0,3577	0,3599	0,3621
1,1	0,3643	0,3665	0,3686	0,3708	0,3729	0,3749	0,3770	0,3790	0,3810	0,3830
1,2	0,3849	0,3869	0,3888	0,3906	0,3925	0,3944	0,3962	0,3980	0,3997	0,4015
1,3	0,4032	0,1049	0,4066	0,4082	0,4099	0,4115	0,4131	0,4147	0,4162	0,4177
1,4	0,4192	0,4207	0,4222	0,4236	0,4251	0,4265	0,4274	0,4292	0,4306	0,4319
1,5	0,4332	0,4345	0,4357	0,4370	0,4382	0,4394	0,4406	0,4418	0,4430	0,4441
1,6	0,4452	0,4463	0,4474	0,4484	0,4495	0,4505	0,4515	0,4525	0,4535	0,4545
1,7	0,4554	0,4564	0,4573	0,4582	0,4591	0,4599	0,4608	0,4616	0,4625	0,4633
1,8	0,4641	0,4648	0,4656	0,4664	0,4671	0,4678	0,4686	0,4693	0,4700	0,4706
1,9	0,4713	0,4719	0,4726	0,4732	0,4738	0,4744	0,4750	0,4756	0,4762	0,4757
2,0	0,4772	0,4778	0,4783	0,4788	0,4796	0,4798	0,4803	0,4808	0,48Ï2	0,4817
2,1	0,4821	0,4826	0,4830	0,4834	0,4838	0,4842	0,4846	0,4850	0,4854	0,4857
2,2	0,4861	0,4864	0,4868	0,4871	0,4874	0,4878	0,4881	0,4884	0,4887	0,4890
2,3	0,4893	0,4896	0,4898	0,4901	0,4903	0,4906	0,4909	0,4911	0,4913	0,4916
2,4	0,4918	0,4920	0,4922	0,4924	0,4927	0,4929	0,4930	0,1932	0,4934	0,4936
2,5	0,4938	0,4940	0,4941	0,4943	0,4945	0,4946	0,4948	0,4949	0,4951	0,4952
2,6	0,4953	0,4955	0,4956	0,4957	0,4958	0,4960	0,4961	0,4962	0,4963	0,4964
2,7	0,4965	0,4966	0,4967	0,4968	0,4969	0,4970	0,4971	0,4972	0,4973	0,4973
2,8	0,4974	0,4975	0,4976	0,4977	0,4977	0,4978	0,4979	0,4980	0,4980	0,4981
2,9	0,4981	0,4982	0,4982	0,4983	0,4984	0,4984	0,4985	0,4985	0,4986	0,4986
3,0	0,4986	0,4986	0,4987	0,4987	0,4988	0,4988	0,4988	0,4989	0,4989	0,4990
3,1	0,4990	0,4990	0,4991	0,4991	0,4991	0,4992	0,4992	0,4992	0,4992	0,4993
3,2	0,4993	0,4993	0,4993	0,4994	0,4994	0,4994	0,4994	0,4994	0,4995	0,4995
3,3	0,4995	0,4995	0,4995	0,4996	0,4996	0,4996	0,4996	0,4996	0,4997	0,4997
3,4	0,4997	0,4997	0,4997	0,4997	0,4997	0,4998	0,4998	0,4998	0,4998	0,4998
3,5	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998
3,6	0,4998	0,4998	0,4998	0,4998	0,4998	0,4998	0,4999	0,4999	0,4999	0,4999
3,7	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999	0,4999
3,8	0,4999	0,4999	0,4999	0,4999	0,4999	0,5000	0,5000	0,5000	0,5000	0,5000
3,9	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000	0,5000

Contents

Educational Edition

Guidelines
for practical tasks of the educational discipline
“Higher and applied mathematics”
for foreign and English-learning full-time students
of the direction 6.140103 “Tourism”

Compiled by **Ie. Misiura**
Editorial director **L. Malyarets**

Editor **L. Novytska**
Proof-reader **L. Novytska**

Навчальне видання

Методичні рекомендації до виконання практичних завдань
з навчальної дисципліни «Теорія ймовірностей та
математична статистика»
для студентів-іноземців та студентів,
що навчаються іноземною мовою,
напряму підговки «Менеджмент» денної форми навчання

Укладач: **Місюра Євгенія Юріївна**
Відповідальний за випуск **Малярець Л. М.**

Редактор Новицька Л.М.
Коректор Новицька Л.М.

План 2016 р. Поз.
Підп. до друку. Формат 60×90 1/16. Папір MultiCopy. Друк на Riso.
Ум.-друк. арк. 2,25. Обл.-вид. арк. Тираж 50 прим. Зам. №

Свідоцтво про внесення до Державного реєстру суб’єктів видавничої справи
Дк №481 від 13.06.2001р.

Видавець і виготівник – видавництво ХНЕУ, 61001, м. Харків, пр. Леніна, 9а