PRACTICE OF TOPIC 7

Task 1. There are **12** students in the group, including **4** A students, **5** B students, the others are C students.

Q1: What is the probability of choosing one A student?

$n = 12$
$m_A = 4$
$m_B = 5$
$m_C = n - m_A - m_B = 12 - 4 - 5 = 3$

$$P(1\ A\ st) = \frac{m_A}{n} = \frac{4}{12} = \frac{1}{3}$$

Q2: What is the probability of choosing one C student?

$n = 12$
$m_C = 3$

$$P(1C\ st) = \frac{m_C}{n} = \frac{3}{12} = \frac{1}{4}$$

Q3: What is the probability of choosing no A student?

no A st means 1B or 1C

$$P(\text{no } A\ st) = P(1B\ st) + P(1C\ st) = \frac{5}{12} + \frac{3}{12} = \frac{8}{12} = \frac{2}{3}$$

Q4: What is the probability of choosing 2 B students?

$n = 12$
$m_B = 5$

$$P(2B\ st) = \frac{C_5^2}{C_{12}^2} = \frac{10}{66} = \frac{5}{33}$$

$$C_5^2 = \frac{5!}{2!\cdot(5-2)!} = \frac{5!}{2!\cdot 3!} = \frac{3!\cdot 4\cdot 5}{2\cdot 3!} = 10$$

$$C_{12}^2 = \frac{12!}{2!\cdot(12-2)!} = \frac{12!}{2!\cdot 10!} = \frac{10!\cdot 11\cdot 12}{2\cdot 10!} = 66$$

Q5: What is the probability of choosing 3 A students and 2 B students?

$$P(\underbrace{3A\,st \text{ and } 2B\,st}_{2+3=5}) = \frac{C_4^3 \cdot C_5^2}{C_{12}^5} = \frac{4 \cdot 10}{792} = \frac{5}{99}$$

$$C_4^3 = \frac{4!}{3! \cdot (4-3)!} = \frac{4!}{3! \cdot 1!} = \frac{3! \cdot 4}{3! \cdot 1} = 4$$

$$C_{12}^5 = \frac{12!}{5!(12-5)!} = \frac{12!}{5! \cdot 7!} = \frac{7! \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 7!} = 792$$

Q6: What is the probability of choosing 3 A students or 2 B students?

$$P(3A\,st \text{ or } 2B\,st) = P(3A\,st) + P(2B\,st) =$$

$$= \frac{C_4^3}{C_{12}^3} + \frac{C_5^2}{C_5^2} = \frac{1}{55} + \frac{5}{33} = \frac{3+25}{165} = \frac{28}{165}$$

Task 2. Melissa collects data on her college graduating class. She finds out that of her classmates, **60 %** are brunettes, **28 %** have blue eyes, and **10 %** are brunettes that have blue eyes. What is the probability that one of Melissa's classmates will be a brunette or have blue eyes.

P(A) = P(brunetts) = 60%=0.6

P(B) = P(blue eyes) = 28%=0.28

P(A and B) = P(both) = 10%=0.1

A and B are ***compatible events****, then P (A or B) = P(A) + P(B) – P(A and B).*

We have P (A or B) = 0.6 + 0.28 – 0.1 = 0.78

Task 3. ***Two students*** **are going to take an exam. The probability that the first student will pass it equals 0,7; for the second one is 0,8.**

What is the probability that

(a) all of the two students will pass the exam;

(b) the first student will only do it;

(c) one student will do it;

(d) no student will do it;

(e) at least one student will do it?

Solution.

Let S_1 be the event that the first student passes the exam,

S_2 be the event that the second student passes the exam,

$\overline{S_1}$ be the event that the first student doesn't pass the exam,

$\overline{S_2}$ be the event that the second student doesn't pass the exam.

Then

$P(S_1)=0{,}7$, $P(\overline{S_1})=1-0{,}7=0{,}3$,

$P(S_2)=0{,}8$, $P(\overline{S_2})=1-0{,}8=0{,}2$.

(A) What is the probability that all of the two students will pass the exam?

$P(A)=P(S_1 \cdot S_2)=P(S_1)\cdot P(S_2)=0{,}7\cdot 0{,}8=0{,}56$.

(B) What is the probability that the first student will only do it?

$P(B)=P(S_1)\cdot P(\overline{S_2})=0{,}7\cdot 0{,}2=0{,}14$.

(C) What is the probability that one student will do it?

$P(C)=P(S_1)\cdot P(\overline{S_2})+P(\overline{S_1})\cdot P(S_2)=0{,}7\cdot 0{,}2+0{,}3\cdot 0{,}8=0{,}14+0{,}24=0{,}38$

(D) What is the probability that no student will do it?

$P(D)=P(\overline{S_1})\cdot P(\overline{S_2})=0{,}3\cdot 0{,}2=0{,}06$

(E) What is the probability that at least one student will do it?

$P(E)=1-P(\overline{E})=1-P(D)=1-0{,}06=0{,}94$

Task 4. In a bolt factory, machines A, B and C manufacture 25%, 35%, 40% respectively. Of the total of their output 5, 4 and 2% are defective.

(A) Find the probability that a randomly drawn bolt is defective.

Solution.

Let A be the event that the drawn bolt is defective.

Let's consider the following complete group of events (hypotheses):

H_A denotes the event that the randomly drawn bolt is made by machine A,

H_B denotes the event that the randomly drawn bolt is made by machine B,

H_C denotes the event that the randomly drawn bolt is made by machine C.

Let's find their probabilities:

$$P(H_A) = 25\% = 0{,}25, \quad P(H_B) = 35\% = 0{,}35, \quad P(H_C) = 40\% = 0{,}4.$$

Since events H_A, H_B and H_C form the complete group, then

$$P(H_A) + P(H_B) + P(H_C) = 0{,}25 + 0{,}35 + 0{,}4 = 1.$$

Let's define conditional probabilities $P(A|H_A)$, $P(A|H_B)$, $P(A|H_C)$:

$$P(A|H_A) = 5\% = 0{,}05, \; P(A|H_B) = 4\% = 0{,}04, \; P(A|H_C) = 2\% = 0{,}02.$$

Here $A|H_A$ is a defective bolt produced by machine A,

$A|H_B$ is a defective bolt produced by machine B,

$A|H_C$ is a defective bolt produced by machine C.

Let's use the total probability formula and find:

$$P(A) = P(H_A)\cdot P(A|H_A) + P(H_B)\cdot P(A|H_B) + P(H_C)\cdot P(A|H_C) =$$
$$= 0{,}25 \cdot 0{,}05 + 0{,}35 \cdot 0{,}04 + 0{,}4 \cdot 0{,}02 = 0{,}0345.$$

We have 3,45 % of defective bolts in the total production.

(B) Find the probability that a randomly drawn bolt is fully functioning.

Solution. Let A be the event that the drawn bolt is defective and $\overline{A}$ be the event that the drawn bolt is fully functioning. Then

$P(\overline{A}) = 1 - P(A) = 1 - 0{,}0345 = 0{,}9655$, i.e.

We have 96,55 % of fully functioning bolts in the total production.

(C) *A bolt is drawn and is found to be defective. What are the probabilities that it was manufactured by machine B?*

Solution. The desired probability of the event $H_B|A$ (the drawn bolt was made by machine B provided it is known that it is defective) is determined by Bayes' formula:

$$P(H_B|A) = \frac{P(H_B)\cdot P(A|H_B)}{P(A)} = \frac{0.35\cdot 0.04}{0.0345} \approx 0{,}406.$$

(D) *A bolt is drawn and is found to be fully functioning. What are the probabilities that it was manufactured by machine A?*

Solution. Let's define conditional probabilities $P(\overline{A}|H_A)$, $P(\overline{A}|H_B)$, $P(\overline{A}|H_C)$ if we know $P(A|H_A) = 5\% = 0{,}05$, $P(A|H_B) = 4\% = 0{,}04$, $P(A|H_C) = 2\% = 0{,}02$.

Then $P(\overline{A}|H_A) = 1 - P(A|H_A) = 1 - 0{,}05 = 0{,}95$,

$P(\overline{A}|H_B) = 1 - P(A|H_B) = 1 - 0{,}04 = 0{,}96$,

$P(\overline{A}|H_C) = 1 - P(A|H_C) = 1 - 0{,}02 = 0{,}98$.

The desired probability of the event $H_A|A$ (the drawn bolt was made by machine A provided it is known that it is fully functioning) is determined by Bayes' formula:

$$P(H_A|\overline{A}) = \frac{P(H_A)\cdot P(\overline{A}|H_A)}{P(\overline{A})} = \frac{0.25\cdot 0.95}{0.9655} \approx 0{,}246.$$