PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Theme: Empirical and logical foundations of probability theory. Elements of combinatorics. Basic theorems of probability theory, their economic interpretation. Probability theory in international trade strategies (part 2)

A classical definition of a probability

So how do we calculate probability?

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

A classical definition of a probability *(класичне визначення ймовірності)*

n is total number of outcomes *(всі можливі результати даного випробування)*

m are outcomes, favorable for an event A *(кількість результатів (фіналів) випробування, в яких настає подія А)*

Sample Space: all the possible outcomes of an experiment.

Example: choosing a card from a deck

There are 52 cards in a deck (not including Jokers)

So the **Sample Space is all 52 possible cards**: {Ace of Hearts, 2 of Hearts, etc... }

Example: Deck of Cards

- the 5 of Clubs is a sample point
- the King of Hearts is a sample point

PROBABILITY ADDITION THEOREMS

- The events are called ***compatible*** (***not mutually exclusive***) if they can occur together in the same experiment.
- The events are called ***incompatible*** (***mutually exclusive***) if they cannot occur together in the same experiment.

Compatible events

(сумісні події)

Incompatible events

(несумісні події)

Probability addition theorem for incompatible events

The probability of realization of at least one of two events ***A*** and ***B*** is given by the formula:

$$P(A+B)=P(A\,or\,B)=P(A)+P(B)$$

where ***A*** and ***B*** are incompatible events.

Example. Ann rolls a die once. a) What is the probability she rolls a 3 and a 6? b) What is the probability she rolls a 3 or a 6?

Solution. a) When one die is rolled, the event ***A*** of rolling a 3 and the event ***B*** of rolling a 6 are events that cannot both happen at the same time, and are called mutually exclusive events. So the probability of rolling a 3 and a 6 is impossible on one roll of a die, and equal to zero, i.e. $P(A and B) = 0$

b) The probability of rolling a 3 (***A***) or a 6 (***B***) is also mutually exclusive event and is calculated by the formula:

$$P(A+B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

TASK

In a pencil-box there are 12 pencils of an equal form and size? Four colored pencils, 8 lead pencils. If the pencil-box is opened, a pencil is taken out, what is the probability that the pencil is

(1) lead? (2) colored? (3) lead and colored?

(4) lead or colored?

Probability addition theorem for compatible events

The probability of realization of at least one of two events ***A*** and ***B*** is given by the formula:

$$P(A+B) = P(A \, or \, B) = P(A) + P(B) - P(A \cap B)$$

where ***A*** and ***B*** are compatible events.

Example. Ann rolls a die once. What is the probability she rolls a prime number or an odd number?

Solution. When one die is rolled, the event ***A*** of rolling a prime or the event ***B*** of rolling an odd number are events that can both happen at the same time, and they are compatible events.

We have $A = \{2, 3, 5\}$, $B = \{1, 3, 5\}$ and obtain

$$P(A) = \frac{3}{6} \qquad P(B) = \frac{3}{6}$$

We find

$$A \cap B = \{3, 5\} \qquad P(A \cap B) = \frac{2}{6}$$

and use the formula:

$$P(A+B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$$

Question 1

There are 12 examination cards on the table numbered from 1 to 12.

What is the probability that the number of randomly taken card will be

(1) prime?

(2) composite?

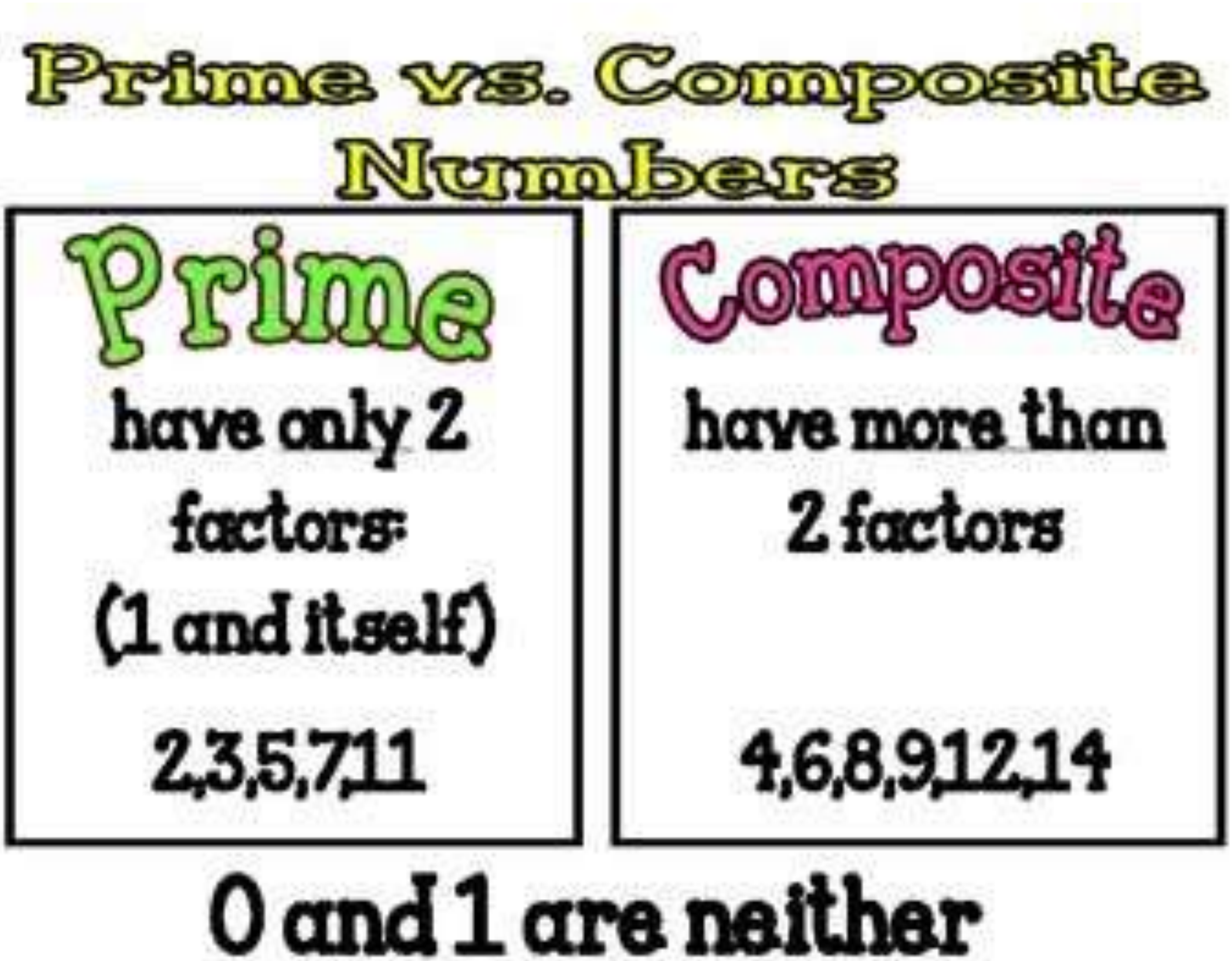
(3) prime or composite?

(4) multiple of 2?

(5) multiple of 3?

(6) multiple of 2 or 3?


Prime Numbers
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100
A Prime Number is a whole number
with only 2 factors (1 and itself).
Composite Numbers
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100
A Composite Number is a whole number
with 3 or more factors.


Prime vs. Composite Numbers
Prime
have only 2 factors: (1 and itself)
2,3,5,7,11
Composite
have more than 2 factors
4,6,8,9,12,14
0 and 1 are neither

TASK

There are 32 examination cards on the table. What is the probability that the number of randomly taken card will be

(1) multiple of 7? (2) multiple of 4?

(3) multiple of 7 or 4? (4) multiple of 7 and 4?

MULTIPLICATION PROBABILITY THEOREMS

- The events are called ***independent*** if the occurrence of one of them does not change the probability of the occurrence of the other one.
- The events are called ***dependent*** if the probability of each of them is changed in connection with the occurrence or nonoccurrence of the other one.

Independent events

незалежні події

Dependent events

залежні події

Multiplication theorem for independent events. When the outcome of one event has no effect on the outcome of another event, we say that the two events are independent events. To obtain the probability of independent events we multiply the probabilities of the separate events, i.e.

$$P(A\cdot B)=P(A\,and\;B)=P(A)\cdot P(B)$$

where ***A*** and ***B*** are independent events.

Example. A coin is tossed and a die is rolled. What is the probability of obtaining a head and a prime number?

Example. A coin is tossed and a die is rolled. What is the probability of obtaining a head and a prime number?

Solution.

The result of tossing a coin cannot possibly affect the outcome of rolling a die. In other words, if the coin landed as a head, it would not affect the way the die would land. Then the outcomes are independent events.

The probability of ***A*** (tossing a head) is $\frac{1}{2}$,

i.e. $P(A) = \frac{1}{2}$

and the probability of ***B*** (rolling a prime number with a die) is $\frac{3}{6}$,

i.e. $P(B) = \frac{3}{6}$

because there are three numbers 2, 3, and 5 that are prime.

Let’s use the formula

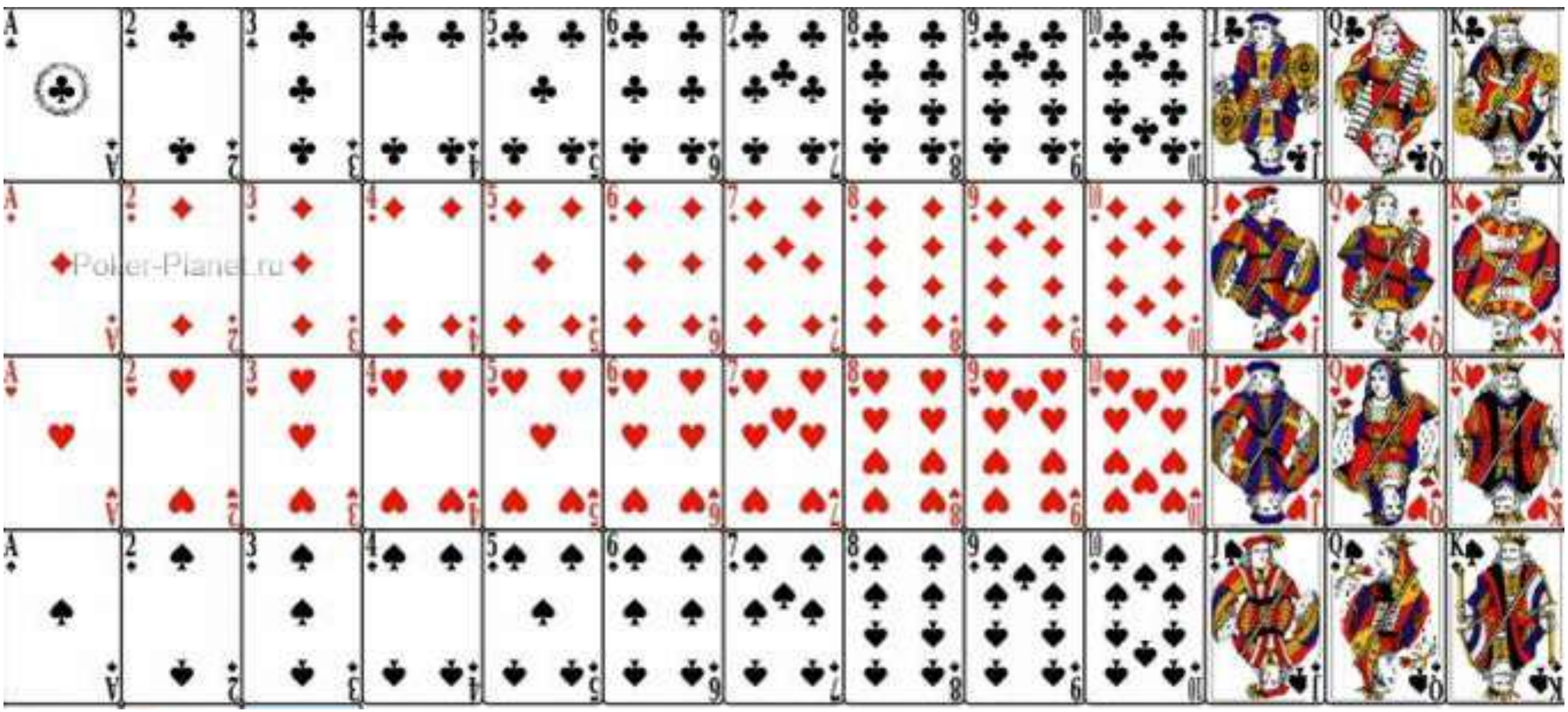
$$P(A\cdot B)=P(A\ and\ B)=P(A)\cdot P(B)$$

and obtain

$$P(A\cdot B)=P(A)\cdot P(B)=\frac{1}{2}\cdot\frac{3}{6}=\frac{1}{4}$$

Example. A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and then an eight?

A DECK OF 52 CARDS



SUITS

SUITS

DIAMOND

SUITS

HEART

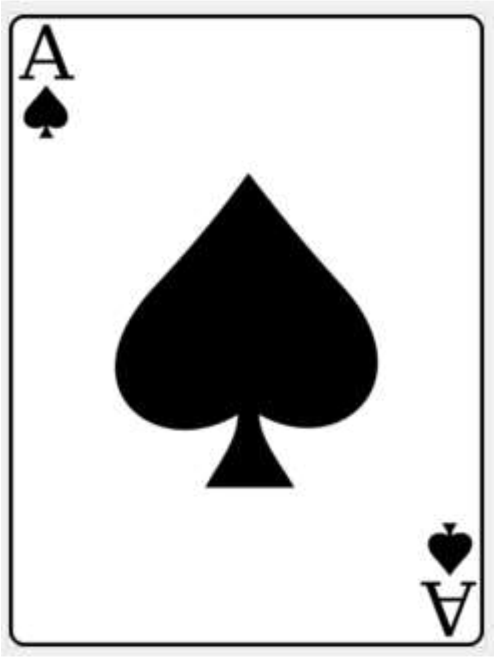
SUITS

CLUB

SUITS

SPADE

AN ACE



A KING

A QUEEN

A JACK


CLIPARTO
CLIPARTO
CLIPARTO
CLIPARTO
CLIPARTO
CLIPARTO
CLIPARTO

Example. A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and then an eight?

Solution.

$$P(A) = P(\text{jack on first pick}) = \frac{4}{52}$$

A JACK

Example. A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and then an eight?

Solution.

$$P(A) = P(\text{jack on first pick}) = \frac{4}{52}$$

$$P(B \mid A) = P(8 \text{ on 2nd pick given jack on 1st pick}) =$$

Example. A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and then an eight?

Solution.

$$\mathrm{P}(\mathrm{A}) = \mathrm{P}(\text{jack on first pick}) = \frac{4}{52}$$

$$\mathrm{P}(\mathrm{B} \mid \mathrm{A}) = \mathrm{P}(8 \text{ on 2nd pick given jack on 1st pick}) = \frac{4}{52} = P(B)$$

The probability that the first card is a queen is 4 out of 52. If the first card is replaced, then the second card is chosen from only 52 cards. Accordingly, the probability that the second card is a jack given that the first card is a queen is 4 out of 52.

Example. A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and then an eight?

Solution.

$$\mathrm{P}(\mathrm{A}) = \mathrm{P}(\text{jack on first pick}) = \frac{4}{52}$$

$$\mathrm{P}(\mathrm{B} \mid \mathrm{A}) = \mathrm{P}(8 \text{ on 2nd pick given jack on 1st pick}) = \frac{4}{52} = P(B)$$

$$\mathrm{P}(\mathrm{A} \cdot \mathrm{B}) = \mathrm{P}(\text{jack and } 8) = \frac{4}{52} \cdot \frac{4}{52} = \frac{16}{2702} = \frac{1}{169}$$

HOMEWORK

A school survey found that 9 out of 10 students like pizza. If three students are chosen at random with replacement, what is the probability that all three students like pizza?

Multiplication theorem for dependent events. If ***A*** and ***B*** are dependent events, then:

$$P(A \cdot B) = P(A\; and\; B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

where $P(B|A)$ or $P_A(B)$ is called the conditional probability of the event ***B*** given the event ***A*** (it means the probability that the event ***B*** will occur given that the event ***A*** has already occurred).

Example. There are 3 nonstandard electric bulbs among 50 electric ones. What is the probability that 2 electric bulbs taken at a time are nonstandard?

Example. There are 3 nonstandard electric bulbs among 50 electric ones. What is the probability that 2 electric bulbs taken at a time are nonstandard?

Solution.

The probability of the event ***A*** that the first

bulb is nonstandard equals

Example. There are 3 nonstandard electric bulbs among 50 electric ones. What is the probability that 2 electric bulbs taken at a time are nonstandard?

Solution.

The probability of the event ***A*** that the first bulb is nonstandard equals $\frac{3}{50}$.

The probability of the second bulb is non-standard (the event ***B***) on conditions that the first bulb is nonstandard (the event ***A***)

equals

The probability of the second bulb is non-standard (the event ***B***) on conditions that the first bulb is nonstandard (the event ***A***)

equals $\frac{2}{49}$

The probability of the second bulb is non-standard (the event ***B***) on conditions that the first bulb is nonstandard (the event ***A***)

equals $\frac{2}{49}$,

because the total number of bulbs and the number of nonstandard bulbs are decreased by 1.

According to the formula

$$P(A\cdot B)=P(A\,and\,B)=P(A)\cdot P(B|A)$$

we have

$$P(A\cdot B)=P(A)\cdot P(B|A)=\frac{3}{50}\cdot\frac{2}{49}\approx 0.0024$$

Example. A card is chosen at random from a pack of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

Example. A card is chosen at random from a pack of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

Solution.

$$\mathrm{P}\left(\mathrm{A}\right) = \mathrm{P}(\text{queen on first pick}) = \frac{4}{52}$$

Example. A card is chosen at random from a pack of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

Solution.

$$P(A) = P(\text{queen on first pick}) = \frac{4}{52}$$

$$P(B \mid A) = P(\text{jack on 2nd pick given queen on 1st pick}) =$$

Example. A card is chosen at random from a pack of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

Solution.

$$P(A) = P(\text{queen on first pick}) = \frac{4}{52}$$

$$P(B \mid A) = P(\text{jack on 2nd pick given queen on 1st pick}) = \frac{4}{51}$$

The probability of choosing a jack on the second pick given that a queen was chosen on the first pick is *a conditional probability*

Example. A card is chosen at random from a pack of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

Solution.

$$P(A) = P(\text{queen on first pick}) = \frac{4}{52}$$

$$P(B \mid A) = P(\text{jack on 2nd pick given queen on 1st pick}) = \frac{4}{51}$$

$$P(A \cdot B) = P(\text{queen and jack}) = \frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$$

HOMEWORK

Three cards are chosen at random from a pack of 52 cards without replacement. What is the probability of choosing 3 aces?

Two random events ***A*** and ***B*** are said to be ***independent*** if the conditional probability of ***A*** given ***B*** coincides with the unconditional probability of ***A***, i.e.

$$P\left(A|B\right) = P(A)$$

A ***conditional probability*** from the formula

$$P(A\cdot B)=P(A\,and\;B)=P(A)\cdot P(B|A)$$

is expressed as:

$$P(B|A)=\frac{P(A\cdot B)}{P(A)}$$

Conditional probability

умовна ймовірність

Example. The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

Example. The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

Solution. Let's denote that

it is Friday as the event ***A***

and

a student is absent as the event ***B***.

According to the condition of this example we have:

the probability that it is Friday (***A***) and that a student is absent (***B***) is 0.03, i.e.

$$P(A \cdot B) = 0.03$$

the probability that it is Friday (***A***) is 0.2, i.e.

$$P(A) = 0.2$$

Then the event that a student is absent (***B***) given that today is Friday (***A***) is denoted by $B|A$

Then the event that a student is absent (***B***) given that today is Friday (***A***) is denoted by $B|A$

Let's find $P(B|A)$ using the formula

$$P(B|A)=\frac{P(A\cdot B)}{P(A)}=\frac{0.03}{0.2}=0.15$$

HOMEWORK

A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

A complete group of events

One says that events $A_1, A_2, \quad , A_n$ form ***a complete group*** of pairwise incompatible events (or mutually exclusive), if exactly one of them necessarily occurs for each realization of the experiment and no other event can occur.

If events $A_1, A_2, \quad , A_n$ form a complete group of pairwise incompatible events, then

$$P(A_1)+P(A_2)+ \quad +P(A_n)=1$$

A complete group of events \ повна група подій

Example

ROLLING A DIE (6 outcomes)

The 1-st event is number 1 $H_1 = \{1\}$

The 2-nd event is number 2 $H_2 = \{2\}$

The 3-rd event is number 3 $H_3 = \{3\}$

The 4-th event is number 4 $H_4 = \{4\}$

The 5-th event is number 5 $H_5 = \{5\}$

The 6-th event is number 6 $H_6 = \{6\}$

Example

ROLLING A DIE (6 outcomes)

The 1-st event is number 1 $H_1 = \{1\}$

The 2-nd event is number 2 $H_2 = \{2\}$

The 3-rd event is number 3 $H_3 = \{3\}$

The 4-th event is number 4 $H_4 = \{4\}$

The 5-th event is number 5 $H_5 = \{5\}$

The 6-th event is number 6 $H_6 = \{6\}$

$$P(H_1) + P(H_2) + \ldots + P(H_6) = \frac{1}{6} + \ldots + \frac{1}{6} = 1$$

For example, two opposite (complementary) events A and $\overline{A}$ form a complete group of incompatible events.

Example

TOSSING A COIN (2 outcomes)

The event ***A*** is head (the 1-st side of a coin)

$$A = \{h\}$$

The event ***not A*** is tail (the 2-nd side of a coin)

$$\overline{A} = \{t\}$$

Example

TOSSING A COIN (2 outcomes)

The event ***A*** is head (the 1-st side of a coin)

$$A = \{h\}$$

The event ***not A*** is tail (the 2-nd side of a coin)

$$\overline{A} = \{t\}$$

$$P(A) = \frac{1}{2} \qquad P(\overline{A}) = \frac{1}{2} \qquad P(A) + P(\overline{A}) = \frac{1}{2} + \frac{1}{2} = 1$$

Example. Let the probability that the shooter scores 10 points, when hitting the target, equals 0.4, 9 points – 0.2, 8 points – 0.2, 7 points – 0.1, 6 points and less – 0.1. What is the probability that the shooter scores no less than 9 points by one short?

Example. Let the probability that the shooter scores 10 points, when hitting the target, equals 0.4, 9 points – 0.2, 8 points – 0.2, 7 points – 0.1, 6 points and less – 0.1. Check whether all events form a complete group of events.

Solution.

Let A_1 be the shooter scoring 10 points,

A_2 be the shooter scoring 9 points,

A_3 be the shooter scoring 8 points,

A_4 be the shooter scoring 7 points,

A_5 be the shooter scoring 6 points and less.

Let A_1 be the shooter scoring 10 points,

A_2 be the shooter scoring 9 points,

A_3 be the shooter scoring 8 points,

A_4 be the shooter scoring 7 points,

A_5 be the shooter scoring 6 points and less.

These events form the complete group of pairwise incompatible events, i.e.

$$P(A_1)+P(A_2)+P(A_3)+P(A_4)+P(A_5)=$$

$$=0.4+0.2+0.2+0.1+0.1=1$$

A NOTION OF AN EVENT INDEPENDENCE

A notion of a pairwise independence of random events

Events $A_1, A_2, \quad , A_n$ are called ***pairwise independent*** if any two events are independent, i.e. $P(A_i \cap A_j) = P(A_i) \cdot P(A_j)$ for any i, j ($i \neq j$).

A pairwise independence of random events \ попарна незалежність випадкових подій

Let events $A_1, A_2, \ldots, A_n$ be independent, ***A*** is ***at least one of n*** events occurs in the experiment. Then $\overline{A}$ is this event that no one of ***n*** events occurs in the experiment, i.e.

$\overline{A} = \overline{A_1} \cdot \overline{A_2} \cdot \ldots \cdot \overline{A_n}$. The events ***A*** and $\overline{A}$ form a complete group of incompatible events, therefore,

$$P(A) = 1 - P\left(\overline{A}\right) = 1 - P\left(\overline{A_1}\right) \cdot P\left(\overline{A_2}\right) \cdot \ldots \cdot P\left(\overline{A_n}\right)$$

This formula is called the ***probability that at least one of events occurs***.

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution.

How many students?

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution.

A1 is the first student passes the exam

A2 is the 2-nd student passes the exam

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution.

A1 is the first student passes the exam

not A1 is the first student doesn’t pass the exam

A2 is the second student passes the exam

not A2 is the second student doesn’t pass the exam

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution. What outcomes are possible?

In this experiment we obtain 4 outcomes:

A1 and A2

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution. What outcomes are possible?

In this experiment we obtain 4 outcomes:

A1 and A2

A1 and not A2

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution. What outcomes are possible?

In this experiment we obtain 4 outcomes:

A1 and A2

A1 and not A2

not A1 and A2

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution. What outcomes are possible?

In this experiment we obtain 4 outcomes:

A1 and A2

A1 and not A2

not A1 and A2

not A1 and not A2

Example. Two students are going to take the exam. The probability that the first student passes it equals 90%, for the second is 75%. What is the probability that

1) one of two students passes the exam;

2) at least one of two students does it.

Solution. What outcomes are possible?

In this experiment we obtain 4 outcomes:

A1 and A2

A1 and not A2

not A1 and A2

not A1 and not A2

These are not equally likely events.

Each student can pass the exam or not.

$P(A1)=90\%=0.9$

$P(A2)=75\%=0.75$

$$P(A) + P(\overline{A}) = 1$$

for complementary events

$$P(A)+P(\overline{A})=1$$

for complementary events

$$P(\overline{A})=1-P(A)$$

Each student can pass the exam or not.

P(A1) = 90% = 0.9

P(A2) = 75% = 0.75

P(not A1)= 1 - 0.9 = 0.1

P(not A2)= 1 - 0.75 = 0.25.

QUESTION 1

What is the probability that one of two students passes the exam?

The event A (one of two students passes the exam) has 2 cases.

The first case is A1 and not A2.
The second one is not A1 and A2.

QUESTION 1

What is the probability that one of two students passes the exam?

The event A (one of two students passes the exam) has 2 cases.

The first case is A1 and not A2.
The second one is not A1 and A2.

The events A1 and A2 are independent.

QUESTION 1

What is the probability that one of two students passes the exam?

The event A (one of two students passes the exam) has 2 cases.

P(A)=P(A1)*P(notA2)+P(notA1)*P(A2)=

=0.9*0.25+0.1*0.75=0.225+0.075=0.3=30%

QUESTION 2

What is the probability that at least one of two students passes the exam?

QUESTION 2

What is the probability that at least one of two students passes the exam?

In this experiment we obtain 4 outcomes:

A1 and A2

A1 and not A2

not A1 and A2

not A1 and not A2

QUESTION 2

What is the probability that at least one of two students passes the exam?

In this experiment we obtain 4 outcomes:

A1 and A2

A1 and not A2

not A1 and A2

not A1 and not A2

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The first case is A1 and not A2.

The second one is not A1 and A2.

The third one is A1 and A2.

QUESTION 2

What is the probability that at least one of two students passes the exam?

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The first case is A1 and not A2.

The second one is not A1 and A2.

The third one is A1 and A2.

The events A1 and A2 are independent.

$$P(A)=P(A1)*P(notA2)+P(notA1)*P(A2)+P(A1)*P(A2)=$$

$$=0.9*0.25+0.1*0.75+0.9*0.75=$$

$$=0.225+0.075+0.675=0.975=97.5\%$$

QUESTION 2

What is the probability that at least one of two students passes the exam?

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The first case is A1 and not A2.

The second one is not A1 and A2.

The third one is A1 and A2.

We can solve it using formula $$P(A)+P(\overline{A})=1$$

QUESTION 2

What is the probability that at least one of two students passes the exam?

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The first case is A1 and not A2.

The second one is not A1 and A2.

The third one is A1 and A2.

We can solve it using formula $$P(A) + P(\overline{A}) = 1$$

$$P(A) = 1 - P(\overline{A})$$

QUESTION 2

What is the probability that at least one of two students passes the exam?

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The first case is A1 and not A2.

The second one is not A1 and A2.

The third one is A1 and A2.

The complementary event (not A) is

QUESTION 2

What is the probability that at least one of two students passes the exam?

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The first case is A1 and not A2.

The second one is not A1 and A2.

The third one is A1 and A2.

The complementary event (not A) is no student passes the exam

QUESTION 2

What is the probability that at least one of two students passes the exam?

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The first case is A1 and not A2.

The second one is not A1 and A2.

The third one is A1 and A2.

The complementary event (not A) is no student passes the exam (it means not A1 and not A2).

QUESTION 2

What is the probability that at least one of two students passes the exam?

The event A (at least one of two students passes the exam means one of two students can pass or two students can pass) contains 3 cases.

The complementary event (not A) is no student passes the exam (it means not A1 and not A2).

We have P(not A)=P(not A1)*P(notA2)=0.1*0.25=0.025=2,5%.

The initial probability is P(A)=1-P(notA)=1-0.025=0.975. We can use this formula to simplify your calculations.

HOMEWORK 1

Three students are going to take the exam. The probability that the first student passes it equals 0.9, the second one is 0.75, the third one is 0.6. What is the probability that

a) all of three students pass the exam;

b) only the first student does it;

c) only the second and the third do it;

d) one student does it;

e) no student does it;

f) at least one student does it.

HOMEWORK 2

Three balls are randomly drawn. What is the probability that all three will be white? At least one of them will be white?

A TOTAL PROBABILITY FORMULA

Formula of a total probability

Let’s suppose that a complete group of pairwise incompatible events $H_1, H_2, \quad , H_n$ is given and the unconditional probabilities $P(H_1), P(H_2), \quad , P(H_n)$ as well as the conditional probabilities $P(A|H_1), P(A|H_2), \quad , P(A|H_n)$ of an event A, are known. Then the probability of ***A*** can be determined by the ***total probability formula***

$$P(A) = \sum_{i=1}^{n} P(H_i) \cdot P(A|H_i)$$

Each of the events $H_1, H_2, \quad , H_n$ is called ***hypothesis.***

Example. Three machines produce the same type of product in a factory. The first one gives 200 articles, the second one does 300 articles and the third one does 500 articles. It is known that the first machine produces 1 % of defective articles, the second one does 2 %, the third one does 4 %. What is the probability that an article selected randomly from the total production will be defective?

Example. Three machines produce the same type of product in a factory. The first one gives 200 articles, the second one does 300 articles and the third one does 500 articles. It is known that the first machine produces 1 % of defective articles, the second one does 2 %, the third one does 4 %. What is the probability that an article selected randomly from the total production will be defective?

Solution.

Let A be the event that the chosen article is defective.

Example. Three machines produce the same type of product in a factory. The first one gives 200 articles, the second one does 300 articles and the third one does 500 articles. It is known that the first machine produces 1 % of defective articles, the second one does 2 %, the third one does 4 %. What is the probability that an article selected randomly from the total production will be defective?

Solution.

Let A be the event that the chosen article is defective.

Let's consider the following complete group of events (hypotheses):

H_1 denotes the event that the randomly selected article is made by the first machine,

H_2 denotes the event that the randomly selected article is made by the second machine,

H_3 denotes the event that the randomly selected article is made by the third machine.

Let A be the event that the chosen article is defective.
Let’s consider the following complete group of events (hypotheses):
H_1 denotes the event that the randomly selected article is made by the first machine,
H_2 denotes the event that the randomly selected article is made by the second machine,
H_3 denotes the event that the randomly selected article is made by the third machine.

Let’s find their probabilities:

$$P(H_1)=\frac{200}{200+300+500}=\frac{200}{1000}=0.2$$

$$P(H_2)=\frac{300}{1000}=0.3 \qquad P(H_3)=\frac{500}{1000}=0.5$$

Let's find their probabilities:

$$P(H_1) = \frac{200}{200+300+500} = \frac{200}{1000} = 0.2$$

$$P(H_2) = \frac{300}{1000} = 0.3 \qquad P(H_3) = \frac{500}{1000} = 0.5$$

Since events H_1, H_2 and H_3 form the complete group, then

$$P(H_1) + P(H_2) + P(H_3) = 0.2 + 0.3 + 0.5 = 1$$

A is the event that the chosen article is defective.

Hypotheses:

H_1 denotes the event that the randomly selected article is made by the first machine,

H_2 denotes the event that the randomly selected article is made by the second machine,

H_3 denotes the event that the randomly selected article is made by the third machine.

Let's define conditional probabilities

$$P(A|H_1),\ P(A|H_2),\ P(A|H_3)$$

Here $A|H_1$ is defective article produced by the first machine,

$A|H_2$ is defective article produced by the second machine,

$A|H_3$ is defective article produced by the third machine.

Let’s use the total probability formula and find:

$$P(A) = P(H_1)\cdot P(A|H_1) + P(H_2)\cdot P(A|H_2) + P(H_3)\cdot P(A|H_3) =$$

$$= 0.2\cdot 0.01 + 0.3\cdot 0.02 + 0.5\cdot 0.04 = 0.028$$

We have 2.8 % of defective articles from the total production.

Example. Three machines produce the same type of product in a factory. The first one gives 200 articles, the second one does 300 articles and the third one does 500 articles. It is known that the first machine produces 1 % of defective articles, the second one does 2 %, the third one does 4 %. What is the probability that an article selected randomly from the total production will be defective?

EXPLANATION

What is the probability that an article selected randomly from the total production will be defective?

The 1-st machine: 200*1%=200*0.01=2 (articles)
The 2-nd machine: 300*2%=300*0.02=6 (articles)
The 3-rd machine: 500*4%=500*0.04=20 (articles)

The total number of defective articles is 2+6+20=28.

The total number of all articles is 200+300+500=1000

Probability of a defective article is $P(A)=\frac{m}{n}=\frac{28}{1000}$

BAYES' FORMULA

Bayes' formula

If it is known that the event A has occurred but it is unknown which of the events $H_1, H_2, \quad , H_n$ has occurred, then ***Bayes' formula*** is used:

$$P(H_k|A) = \frac{P(H_k) \cdot P(A|H_k)}{P(A)} \qquad k = 1, 2, \quad , n$$

and $P(H_1|A) + P(H_2|A) + \quad + P(H_n|A) = 1$

where $P(H_i|A)$ is called ***a posteriori probability*** *(final probability)*.

A priori probability \ **апріорна ймовірність**

A posteriori probability (final probability) \ **апостеріорна ймовірність**

Example. Let’s use the condition of **the previous example** and solve the following problem. It is known that a selected article is defective. What is the probability that this article was made by the second machine?

Example. Let’s use the condition of **example 8** and solve the following problem. It is known that a selected article is defective. What is the probability that this article was made by the second machine?

Solution. The desired probability of the event $H_2|A$ (the selected article was made by the second machine under condition that it is known that it is defective) is determined by Bayes' formula

$$P(H_2|A) = \frac{P(H_2)\cdot P(A|H_2)}{P(A)} = \frac{0.3\cdot 0.02}{0.028} = \frac{6}{28} = \frac{3}{14}$$

EXPLANATION

What is the probability that this selected defective article was made by the second machine?

The 1-st machine: 200*1%=200*0.01=2 (articles)
The 2-nd machine: 300*2%=300*0.02=6 (articles)
The 3-rd machine: 500*4%=500*0.04=20 (articles)

The total number of defective articles is 2+6+20=28.

The total number of all articles is 200+300+500=1000

Probability of a defective article from the second machine:

$$P(H_2|A)=\frac{6}{28}=\frac{3}{14}$$

A TOTAL PROBABILITY FORMULA

Формула повної ймовірності

BAYES' FORMULA /

формула Байєса

Question 2

Three machines produce the same type of product in a factory. The first one gives 40 articles, the second one does 60 articles and the third one does 100 articles. It is known that the first machine produces 95 % of standard articles, the second one does 98 %, the third one does 97 %.

(1) What is the probability that an article selected randomly from the total production will be standard?

(2) What is the probability that an article selected randomly from the total production will be defective?

Question 3

The exercise is solved by 20 students, including 6 A students, 10 B students, the others are C students. The probability that the A student solves the exercise equals 0.8, the B student solves the exercise equals 0.7, the C student solves the exercise equals 0.5.

1) What is the probability that the exercise is solved?

2) It is known that the exercise has been solved. What is the probability that the exercise has been solved by the B student?

Question 4

The right pocket contains three coins by 50 cent and 4 coins by 25 cent, the left pocket contains 6 coins by 50 cent and 3 coins by 25 cent. One coin is moved out of the right pocket into the left one.

1) What is the probability of moving out of the left pocket a coin by 50 cent at random?

2) What is the probability of moving out of the left pocket a coin by 25 cent at random?

ECONOMIC MEANING

Importance of Probability:

The concept of probability is of great importance in everyday life. Statistical analysis is based on this valuable concept. In fact the role played by probability in modern science is that of a substitute for certainty.

ECONOMIC MEANING

The following discussion explains it further:

a) The probability theory is very much helpful for making prediction. ***Estimates*** and ***predictions*** form an important part of research investigation. With the help of statistical methods, we make estimates for the further analysis. Thus, statistical methods are largely dependent on the theory of probability.

ECONOMIC MEANING

The following discussion explains it further:

b) It has also immense importance in decision making.

c) It is concerned with the planning and controlling and with the occurrence of accidents of all kinds.

ECONOMIC MEANING

The following discussion explains it further:

d) It is one of the inseparable tools for all types of formal studies that involve uncertainty.

e) The concept of probability is not only applied in business and commercial lines, rather than it is also applied to all scientific investigation and everyday life.

Thank You for your time and attention! Hope this presentation was educational and helpful to you ☺