Example 1

Task 1. The function is given as:

$$f(x) = -\frac{1}{3}x^3 + 3x^2 - 5x - 1$$

Let's find the derivative:

STEP 1.	USE OCTAVE https://octave-online.net/

$f'(x) = \left(-\frac{1}{3}x^3 + 3x^2 - 5x - 1\right)' = -\frac{1}{3}\cdot 3x^2 + 3\cdot 2x - 5 - 0 = -x^2 + 6x - 5$	octave:1> x=sym("x")
	x = (sym) x
	
	octave:2> f=-1/3*x^3+3*x^2-5*x-1
	f = (sym)
	
	   3
	  x 2
	- ── + 3·x - 5·x - 1
	  3
	
	octave:3> df=diff(f,x)
	df = (sym)
	
	   2
	- x + 6·x - 5

Let’s find the critical points or solve this equation $f'(x)=0$ and find its roots:

$$f'(x) = -x^2 + 6x - 5 = 0$$

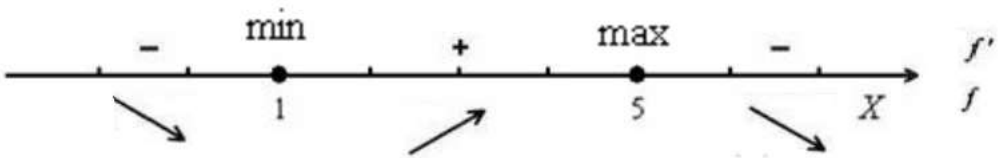
$$D = 36 - 20 = 16 > 0$$

$$\sqrt{D} = \sqrt{16} = 4$$

$$x = \frac{-6+4}{-2} = 1$$

$$x = \frac{-6-4}{-2} = 5$$

Let’s find intervals of increasing and decreasing:



Checking:

$$f'(0) = -0^2 + 6 \cdot 0 - 5 = -5 < 0,$$

$$f'(2) = -2^2 + 6 \cdot 2 - 5 = -4 + 12 - 5 = 3 > 0,$$

$$f'(6) = -6^2 + 6 \cdot 6 - 5 = -36 + 36 - 5 = -5 < 0,$$

Calculate coordinates of max and min:

$\min f(x)= f(1)=-\frac{1}{3}\cdot 1^3+3\cdot 1^2-5\cdot 1-1=-\frac{1}{3}+3-5-1=-\frac{10}{3}=-3\frac{1}{3}$
$\max f(x)= f(5)=-\frac{1}{3}\cdot 5^3+3\cdot 5^2-5\cdot 5-1=-\frac{125}{3}+75-25-1=\frac{22}{3}=7\frac{1}{3}$
i.e. the point of maximum is $\left(5;7\frac{1}{3}\right)$ and the point of minimum is $\left(1;-3\frac{1}{3}\right)$
	`octave:4> fmin=limit(f,1)`
	`fmin = (sym) -10/3`
	`octave:5> fmax=limit(f,5)`
	`fmax = (sym) 22/3`

The function increases on $(1;5)$ and decreases on $(-\infty;1)\cup(5;+\infty)$

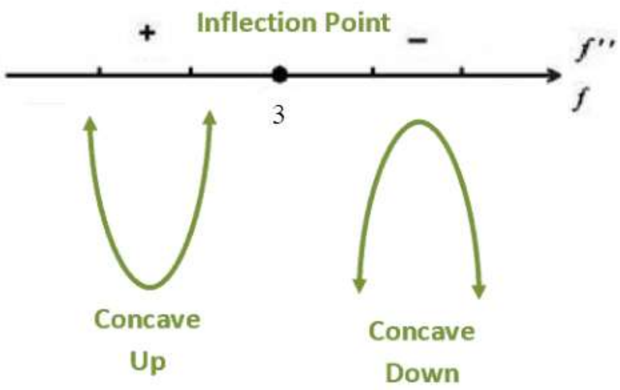
STEP 2.	USE OCTAVE
$f''(x)=(f'(x))'=\left(-x^2+6x+5\right)'=-2x+6=6-2x$	`octave:6> d2f=diff(df,x)` `d2f = (sym) 6 - 2·x`

Let's find the inflection point or solve this equation $f''(x)=0$ and find its root:

$$6-2x=0$$
$$-2x=-6$$
$$x=\frac{-6}{-2}$$
$$x_{\inf l}=3,$$

then $f_{\inf l}(3)=-\frac{1}{3}\cdot 3^3+3\cdot 3^2-5\cdot 3-1=2$, i.e.
the inflection point is $(3;2)$
	USE OCTAVE
	https://octave-online.net/
	`octave:7> f_inlf=-1/3*3^3+3*3^2-5*3-1`
	`f_inlf = 2`

Let's find intervals of concavity up and concavity down:



Checking:

$f''(1) = 6 - 2 \cdot 1 = 6 - 2 = 4 > 0$

$f''(4) = 6 - 2 \cdot 4 = 6 - 8 = -2 < 0$

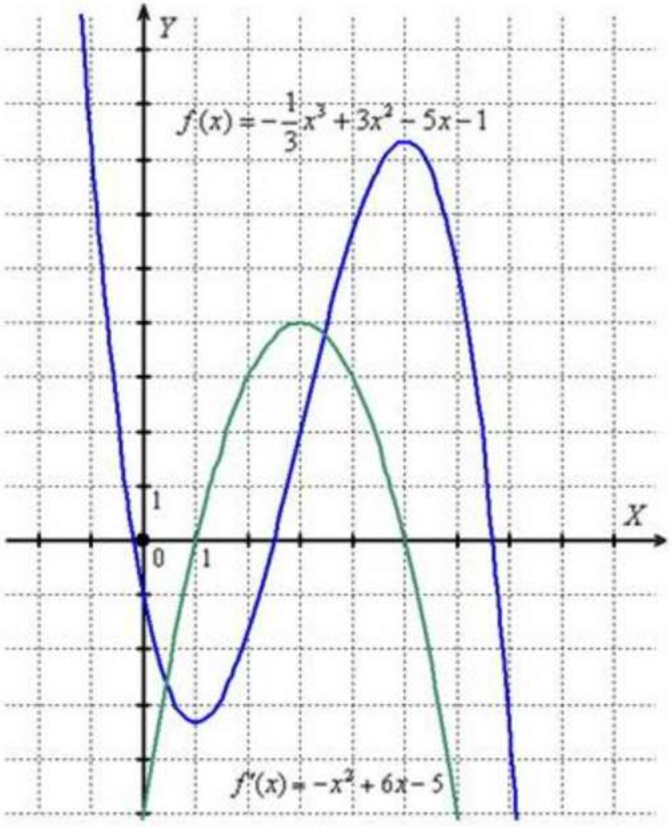
The function is concave up on $(-\infty; 3)$ and concave down on $(3; +\infty)$.

Checking the extremums:

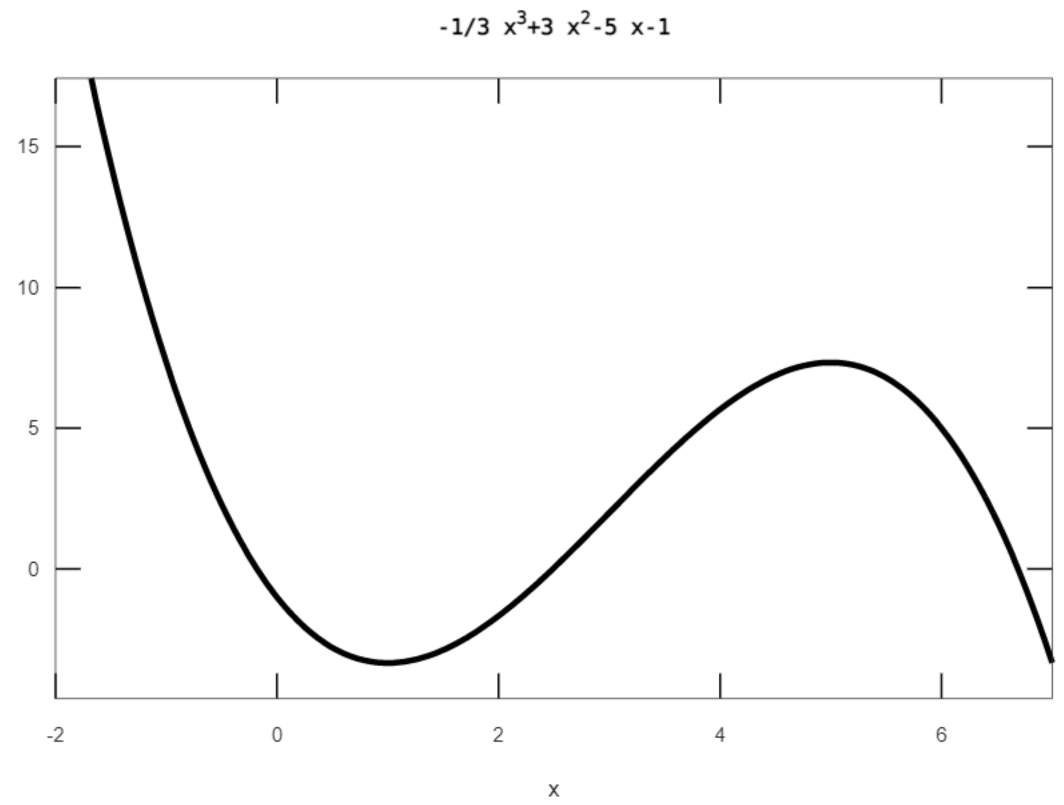
$f''(1) = 6 - 2 \cdot 1 = 6 - 2 = 4 > 0$ (it means $x = 1$ *is* min)

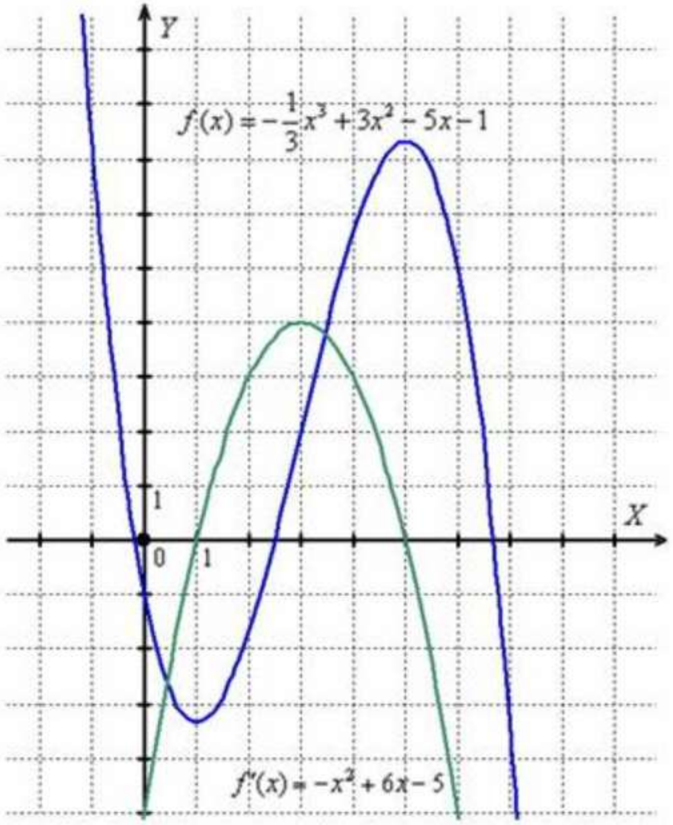
$f''(5) = 6 - 2 \cdot 5 = 6 - 10 = -4 < 0$ (it means $x = 5$ *is* max)

STEP 3.	USE OCTAVE
Let's find the intersection point with y-axis (or $x=0$): $f_{int\_point}(0)=-\frac{1}{3}\cdot 0^3+3\cdot 0^2-5\cdot 0-1=-1$, i.e. we get the intersection point $(0;-1)$	`octave:8> d2f=diff(df,x)` `d2f = (sym) 6 - 2·x`

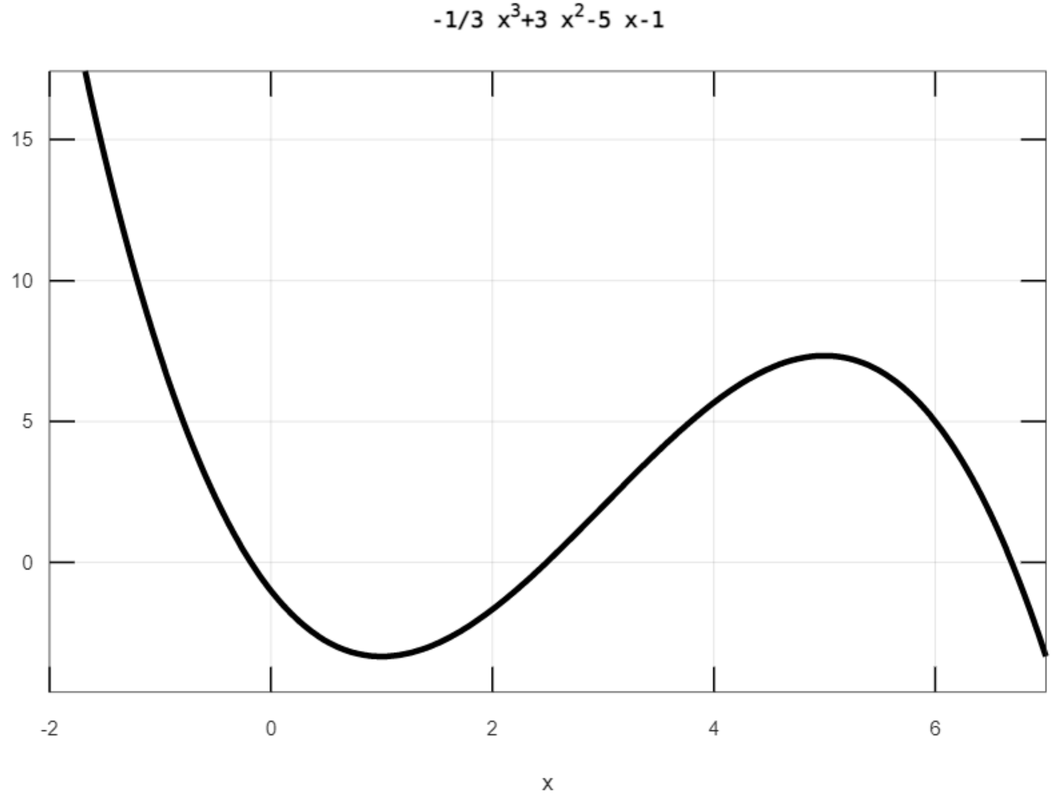


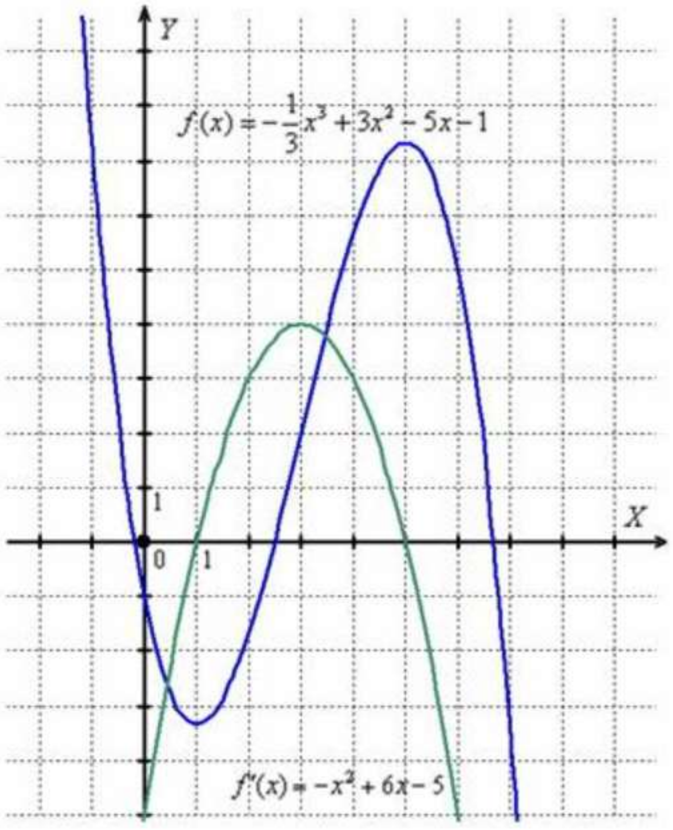

```
octave:9>  L1=ezplot('-1/3*x^3+3*x^2-5*x-1',[-2,7,-10,10])
octave:10> set(L1,'LineWidth',3,'Color','k')
```



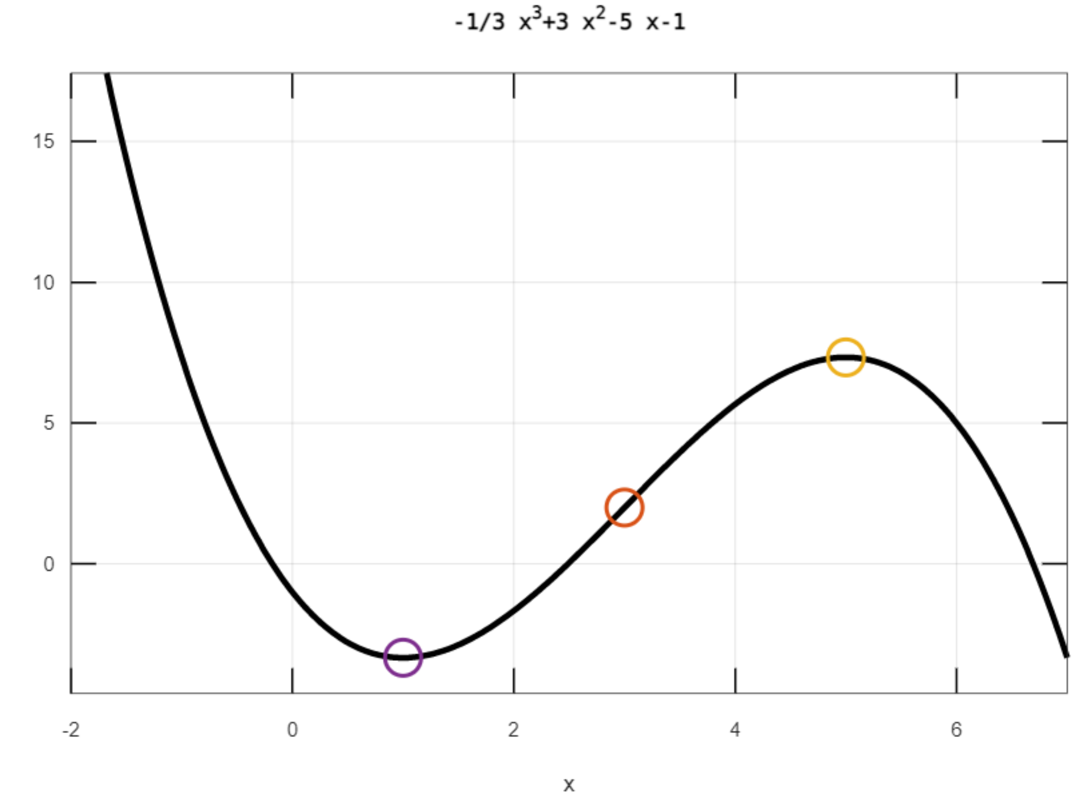


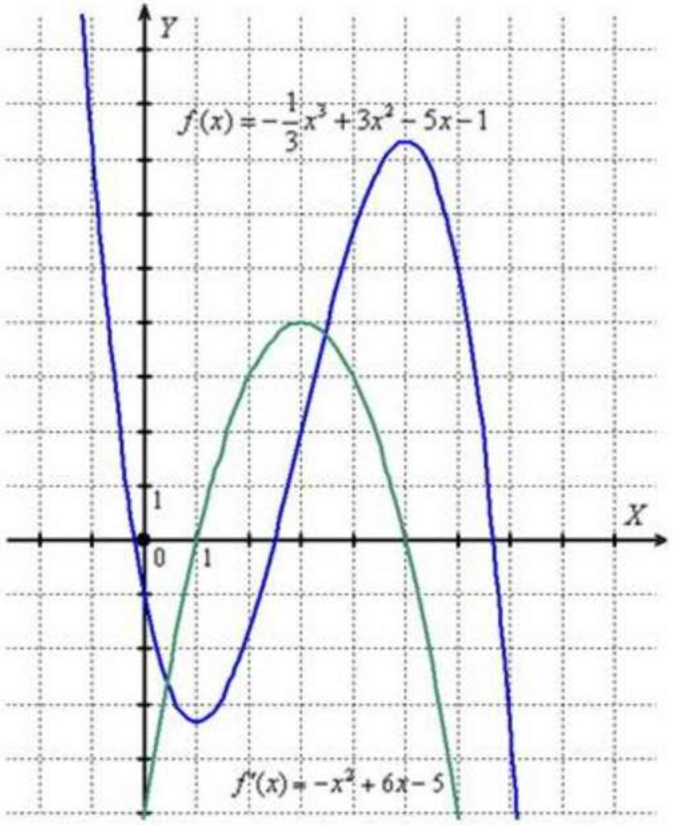

```
octave:11> grid on
```



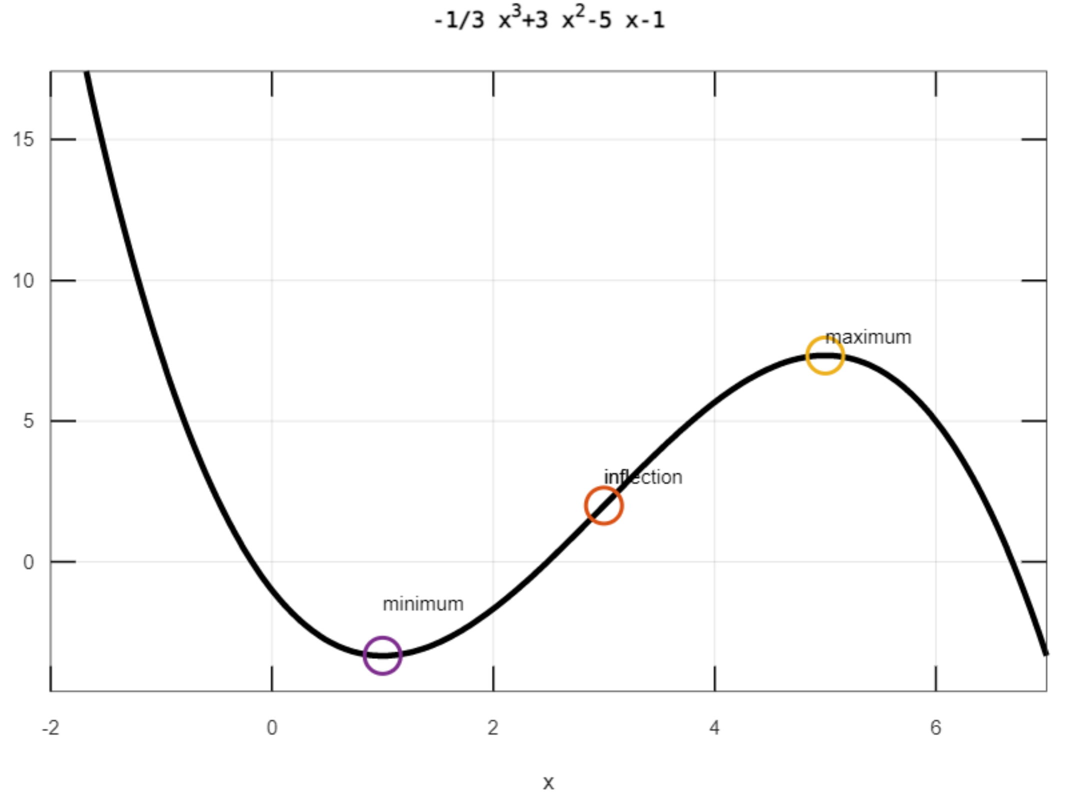


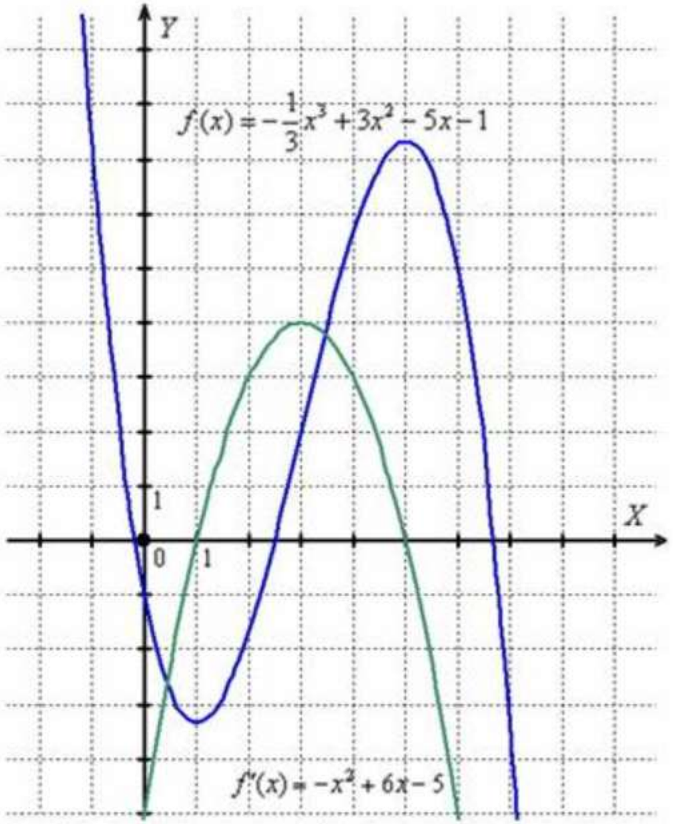

```
octave:12> hold on
octave:13> plot(3,2,'o')
octave:14> plot(5,22/3,'o')
octave:15> plot(1,-10/3,'o')
```



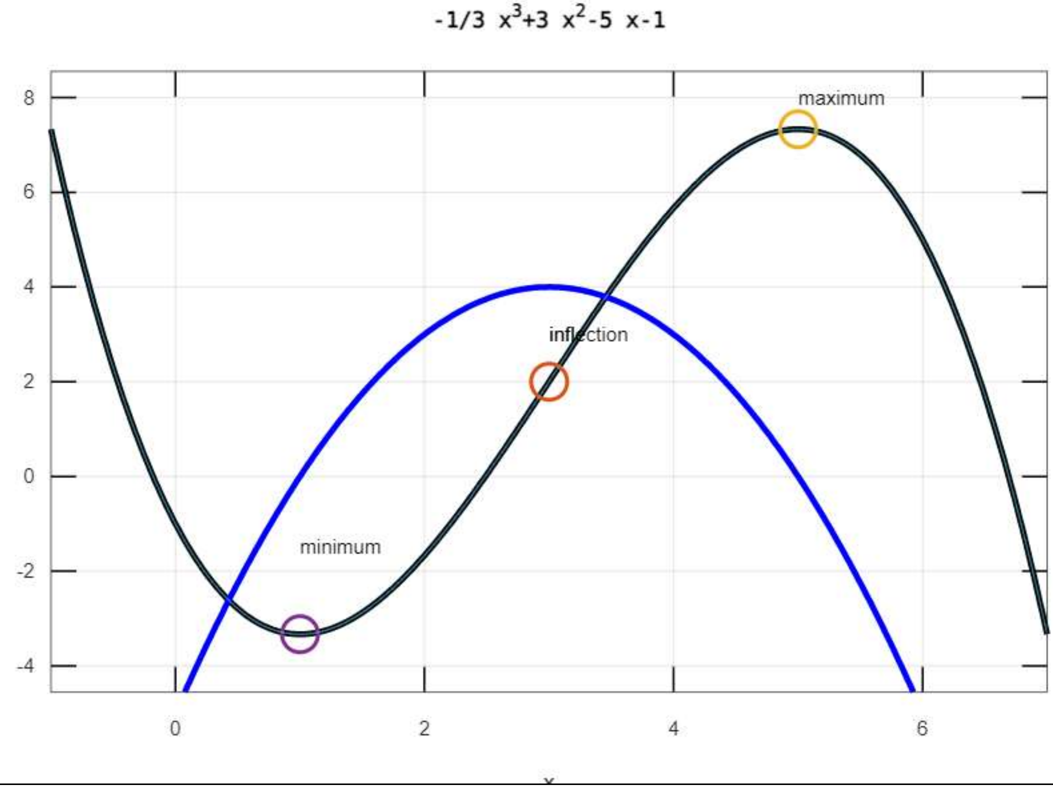



```
octave:16> text(3,3,'inflection')
octave:17> text(1,-1.5,'minimum')
octave:18> text(5,8,'maximum')
```






```
octave:19> L2=ezplot('-x^2+6*x-5',[-1,7,-5,5])
octave:20> set(L2,'LineWidth',3,'Color','b')
```



Example 2

Task 2. The function is given as:

$$f(x)=\frac{4x}{2x+3}$$

Step 1. Let's consider the break point: $x=-\frac{3}{2}$

Let's find the vertical asymptote:

$$\lim_{x\to-\frac{3}{2}-0} f(x)=\lim_{x\to-\frac{3}{2}-0}\frac{4x}{2x+3}=\frac{4\cdot\left(-\frac{3}{2}-0\right)}{2\cdot\left(-\frac{3}{2}-0\right)+3}=\frac{-6}{-3-0+3}=\frac{-6}{-0}=+\infty$$

$$\lim_{x\to-\frac{3}{2}+0} f(x)=\lim_{x\to-\frac{3}{2}+0}\frac{4x}{2x+3}=\frac{4\cdot\left(-\frac{3}{2}+0\right)}{2\cdot\left(-\frac{3}{2}+0\right)+3}=\frac{-6}{-3+0+3}=\frac{-6}{+0}=-\infty$$

Then both limits are infinity, then the vertical asymptote is $x=-\frac{3}{2}$

Step 2. Let's find the oblique (or horizontal) asymptote:

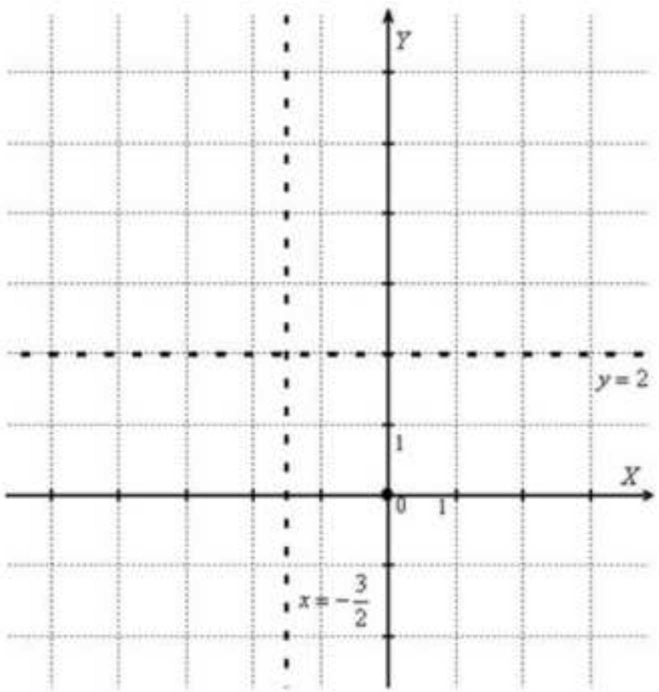
$$k=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{\frac{4x}{2x+3}}{x}=\lim_{x\to\pm\infty}\frac{4x}{x(2x+3)}=\lim_{x\to\pm\infty}\frac{4}{2x+3}=0$$

$$b=\lim_{x\to\pm\infty}(f(x)-kx)=\lim_{x\to\pm\infty}\left(\frac{4x}{2x+3}-0\cdot x\right)=\lim_{x\to\pm\infty}\frac{4x}{2x+3}=\frac{\infty}{\infty}=\lim_{x\to\pm\infty}\frac{\frac{4x}{x}}{\frac{2x+3}{x}}=\lim_{x\to\pm\infty}\frac{4}{2+\frac{3}{x}^{\to 0}}=\frac{4}{2}=2$$

Let’s substitute:

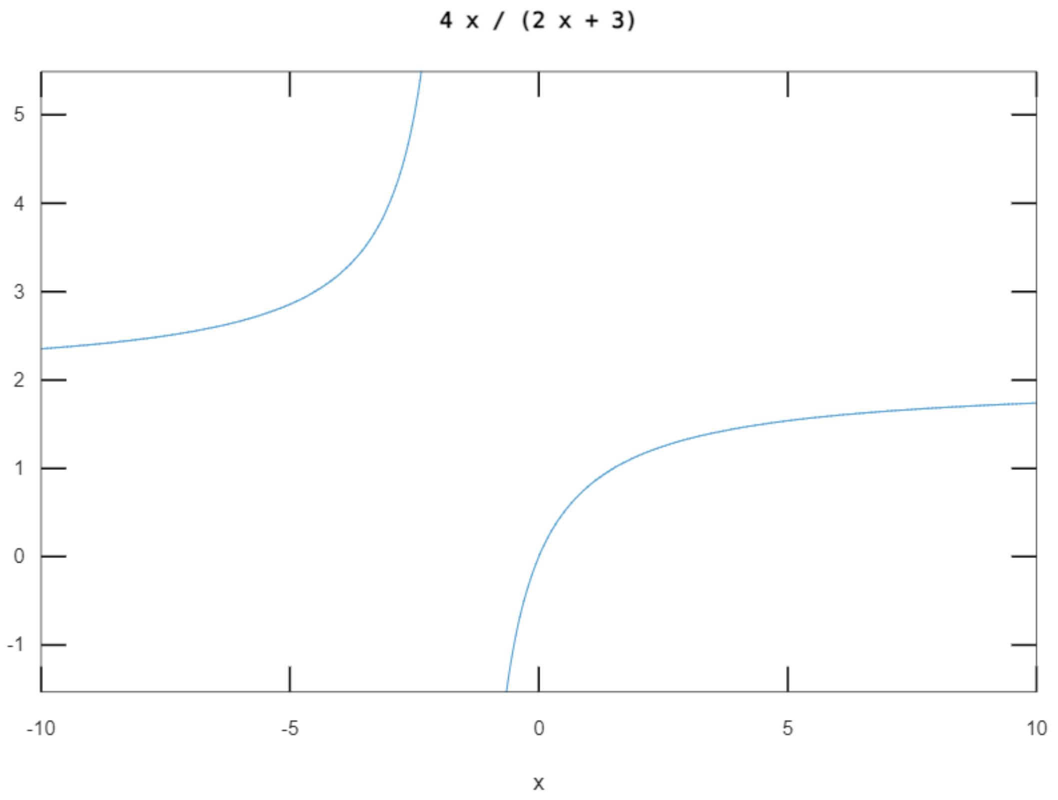
$$y=kx+b$$
$$y=0\cdot x+2$$
$$y=2$$

It’s the horizontal asymptote.

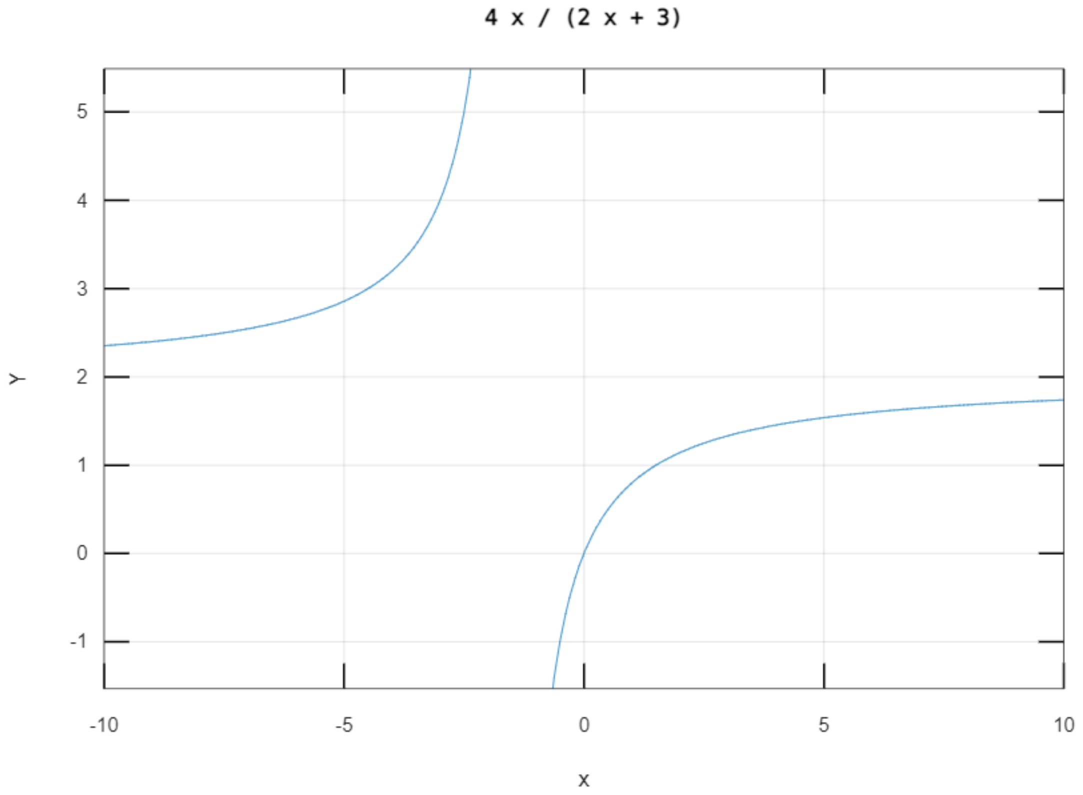


Let's plot the graph:

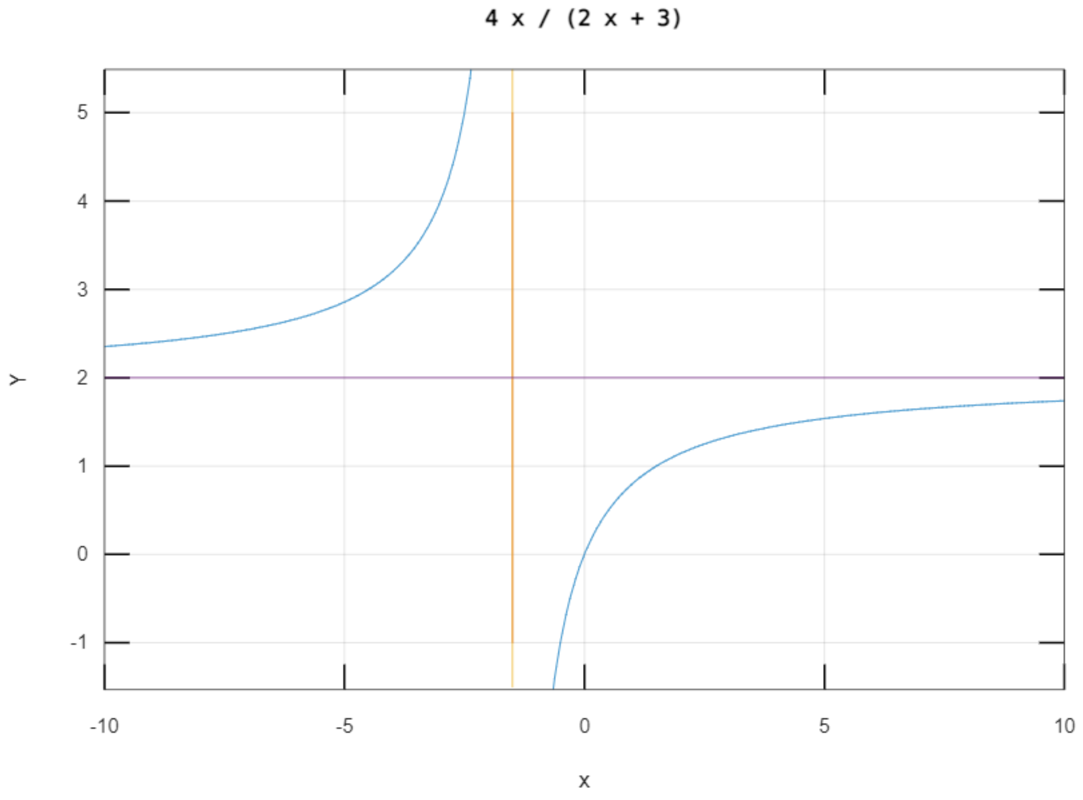
```
octave:1> syms x

octave:2> f=4*x/(2*x+3)
f = (sym)

   4·x
 ───────
 2·x + 3

octave:3> ezplot(f,[-10,10,-10,10])
```



```
octave:4> grid on
octave:5> ylabel('Y')
octave:6> hold on
```



```
octave:7> plot([-3/2 -3/2],[-1.5 5.5])
octave:8> plot([-10 10],[2 2])
```



доцент Місюра Є.Ю. / docent Misiura Ie.

```
octave:9> text(-1,5,'x=-3/2 (vert.as)')
octave:10> text(-8,1.8,'y=2 (horiz.as)')
```