Doc. Misiura Ie.Iu.

DERIVATIVES

Example 1:

$$y' = (3\cos x)' = 3(\cos x)' = 3(-\sin x) = -3\sin x$$

Example 2:

$$y' = \left(6 + x + 3x^2 - \sin x - 2\sqrt[3]{x} + \frac{1}{x^2} - 11ctgx\right)' =$$

$$= (6)' + (x)' + (3x^2)' - (\sin x)' - \left(2x^{\frac{1}{3}}\right)' + \left(x^{-2}\right)' - (11ctgx)' =$$

$$= (6)' + (x)' + 3(x^2)' - (\sin x)' - 2\left(x^{\frac{1}{3}}\right)' + \left(x^{-2}\right)' - 11(ctgx)' =$$

$$= 0 + 1 + 3\cdot 2x - \cos x - 2\cdot\frac{1}{3}\cdot x^{-\frac{2}{3}} + (-2)\cdot x^{-3} - 11\cdot\left(-\frac{1}{\sin^2 x}\right) =$$

$$= 1 + 6x - \cos x - \frac{2}{3\sqrt[3]{x^2}} - \frac{2}{x^3} + \frac{11}{\sin^2 x}$$

Example 3:

$$y' = (x^3 \arcsin x)' = (x^3)' \arcsin x + x^3(\arcsin x)' =$$

$$= 3x^2 \arcsin x + x^3 \cdot \frac{1}{\sqrt{1-x^2}} = 3x^2 \arcsin x + \frac{x^3}{\sqrt{1-x^2}}$$

Example 4:

$$y' = ((x^2 + 7x - 1)\log_3 x)' = (x^2 + 7x - 1)'\log_3 x + (x^2 + 7x - 1)(\log_3 x)' =$$

$$= ((x^2)' + 7(x)' - (1)')\log_3 x + (x^2 + 7x - 1)(\log_3 x)' =$$

$$= (2x + 7\cdot 1 - 0)\log_3 x + (x^2 + 7x - 1)\cdot\frac{1}{x\ln 3} =$$

$$= (2x + 7)\log_3 x + \frac{(x^2 + 7x - 1)}{x\ln 3}$$

Example 5:

$$y' = \left(\frac{2(3x-4)}{x^2+1}\right)' = 2\cdot\left(\frac{3x-4}{x^2+1}\right)' =$$

$$= 2\cdot\left(\frac{(3x-4)'(x^2+1) - (3x-4)(x^2+1)'}{(x^2+1)^2}\right) =$$

$$= 2\cdot\left(\frac{3\cdot(x^2+1) - (3x-4)\cdot 2x}{(x^2+1)^2}\right) = 2\cdot\left(\frac{3x^2 + 3 - 6x^2 + 8x}{(x^2+1)^2}\right) =$$

$$= \frac{2(-3x^2 + 8x + 3)}{(x^2+1)^2}$$

Example 6:

$$y'=\left(x+\frac{1}{\sqrt{x}}-\frac{3}{x}\right)'=(x)'+\left(x^{-\frac{1}{2}}\right)'-3(x^{-1})'=$$

$$=1-\frac{1}{2}\cdot x^{-\frac{3}{2}}-3(-1)x^{-2}=1-\frac{1}{2\sqrt{x^3}}+\frac{3}{x^2}$$

Example 7:

$$y'=(xe^x)'=(x)'e^x+x(e^x)'=1\cdot e^x+xe^x=e^x(x+1)$$

Example 8:

$$y'=(\sin(3x-5))'=\cos(3x-5)\cdot(3x-5)'=\cos(3x-5)\cdot(3-0)=$$
$$=3\cos(3x-5)$$

Example 9:

$$y'=((2x+1)^5)'=5\cdot(2x+1)^4\cdot(2x+1)'=$$
$$=5\cdot(2x+1)^4\cdot(2+0)=10\cdot(2x+1)^4$$

Example 10:

$$y'=(arctg\sqrt{x})'=\frac{1}{1+(\sqrt{x})^2}\cdot(\sqrt{x})'=\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{x}(1+x)}$$

Example 11:

$$y'=(\sqrt{arctgx})'=\frac{1}{2\sqrt{arctgx}}\cdot(arctgx)'=\frac{1}{2(1+x^2)\sqrt{arctgx}}$$

Example 12:

$$y'=(\sqrt[3]{x^2+tgx+15})'=\left(\left(x^2+tgx+15\right)^{\frac{1}{3}}\right)'=$$

$$=\frac{1}{3}\cdot\left(x^2+tgx+15\right)^{-\frac{2}{3}}\cdot(x^2+tgx+15)'=$$

$$=\frac{1}{3\cdot\sqrt[3]{(x^2+tgx+15)^2}}\cdot((x^2)'+(tgx)'+(15)')=$$

$$=\frac{1}{3\cdot\sqrt[3]{(x^2+tgx+15)^2}}\cdot\left(2x+\frac{1}{\cos^2 x}\right)$$

Example 13:

$$y'=\left(-\frac{1}{\cos x}\right)'=-\left(\frac{1}{\cos x}\right)'=-(cos^{-1}x)'=$$

$$=-(-1)\cdot cos^{-2}x\cdot(\cos x)'=\frac{1}{\cos^2 x}\cdot(-\sin x)=-\frac{\sin x}{\cos^2 x}$$

Example 14:

$$y' = (7^{\arcsin^2 x})' = 7^{\arcsin^2 x} \cdot \ln 7 \cdot (\arcsin^2 x)' =$$
$$= 7^{\arcsin^2 x} \cdot \ln 7 \cdot 2\arcsin^1 x \cdot (\arcsin x)' =$$
$$= 7^{\arcsin^2 x} \cdot \ln 7 \cdot 2\arcsin x \cdot \frac{1}{\sqrt{1-x^2}} =$$
$$= \frac{2\ln 7 \cdot 7^{\arcsin^2 x} \cdot \arcsin x}{\sqrt{1-x^2}}$$

Example 15:

$$\left((2x-1)\sqrt{x}\right)' = (2x-1)'\sqrt{x} + (2x-1)\left(\sqrt{x}\right)' =$$
$$= 2\sqrt{x} + (2x-1) \bullet \frac{1}{2\sqrt{x}} = 2\sqrt{x} + \frac{2x-1}{2\sqrt{x}} =$$
$$= \frac{\left(2\sqrt{x}\right)^2 + 2x-1}{2\sqrt{x}} = \frac{4x+2x-1}{2\sqrt{x}} = \frac{6x-1}{2\sqrt{x}}.$$

Example 16:

$$\left(\frac{\sqrt{x}}{2x+1}\right)' = \frac{\left(\sqrt{x}\right)'(2x+1) - \sqrt{x}\,(2x+1)'}{(2x+1)^2} =$$
$$= \frac{\frac{1}{2\sqrt{x}}(2x+1) - \sqrt{x}\,(2)}{(2x+1)^2} = \frac{\frac{2x+1}{2\sqrt{x}} - 2\sqrt{x}}{(2x+1)^2} = \frac{2x+1-\left(2\sqrt{x}\right)^2}{2\sqrt{x}\,(2x+1)^2} =$$
$$= \frac{2x+1-4x}{2\sqrt{x}\,(2x+1)^2} = \frac{1-2x}{2\sqrt{x}\,(2x+1)^2}$$

Example 17:

$$y = 3e^x - 3^x$$
$$y' = 3\left(e^x\right)' - \left(3^x\right) = 3e^x - 3^x \ln 3$$

Example 18:

$$y' = (x^5 - 4x^3 + 2x^2 - 7x)' =$$
$$= (x^5)' - (4x^3)' + (2x^2)' - (7x)' =$$
$$= (x^5)' - 4(x^3)' + 2(x^2)' - 7(x') =$$
$$= 5x^4 - 12x^2 + 4x - 7.$$

Example 19:

$$y' = \left[(1-x^3)(x^4+4x) \right]' =$$
$$= (1-x^3)'(x^4+4x) + (x^4+4x)'(1-x^3) =$$
$$= -3x^2(x^4+4x) + (4x^3+4)(1-x^3) =$$
$$= -7x^6 - 12x^3 + 4.$$

Example 20:

$$y' = (x^2 \ln x)' = (x^2)' \ln x + x^2 (\ln x)'$$
$$y' = 2x \cdot \ln x + x^2 \cdot \frac{1}{x}$$
$$y' = 2x \ln x + x$$

Example 21:

$$(\cos 2x)' = -2\sin 2x$$

$$(\sin 3x)' = 3\cos 3x,$$

$$\left(e^{5x}\right)' = 5e^{5x},$$

$$(\ln 2x)' = \frac{1}{x},$$

$$(\arcsin 4x)' = \frac{4}{\sqrt{1-16x^2}},$$

$$(\mathrm{tg}\, 6x)' = \frac{6}{\cos^2 6x}.$$

Example 22:

$$y' = (2^x - \mathrm{arctg} x)' = (2^x)' - (\mathrm{arctg} x)'$$
$$y' = 2^x \ln 2 - \frac{1}{1+x^2}$$

Example 23:

$$y'(x) = \left(\sin^3 x\right)' = 3\sin^2 x \cdot (\sin x)' = 3\sin^2 x \cos x$$

Example 24

$$y' = (\sin(\operatorname{tg}(\sqrt{x})))' = \cos(\operatorname{tg}(\sqrt{x})) \cdot (\operatorname{tg}(\sqrt{x}))'$$

$$y' = \cos(\operatorname{tg}\sqrt{x}) \cdot \frac{1}{\cos^2\sqrt{x}} \cdot (\sqrt{x})'$$

$$y' = \cos(\operatorname{tg}\sqrt{x}) \cdot \frac{1}{\cos^2\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$y' = \frac{\cos(\operatorname{tg}\sqrt{x})}{2\sqrt{x}\cos^2\sqrt{x}}$$

Example 25

$$\left(\frac{x}{\arccos 3x}\right)' = \frac{x' \cdot \arccos 3x - x \cdot (\arccos 3x)'}{\arccos^2 3x} =$$

$$= \frac{1 \cdot \arccos 3x - x \cdot \left(-\frac{3}{\sqrt{1-9x^2}}\right)}{\arccos^2 3x} = \frac{\sqrt{1-9x^2}\arccos 3x + 3x}{\sqrt{1-9x^2}\arccos^2 3x}$$

Example 26

$$y' = (3^{2x})' \cdot \operatorname{arctg} 5x + 3^{2x} \cdot (\operatorname{arctg} 5x)' = 3^{2x} \cdot \ln 3 \cdot 2 \cdot \operatorname{arctg} 5x + 3^{2x} \cdot \frac{5}{1+25x^2}$$

Example 27

$$y' = \left(\frac{9}{x^3} + \sqrt[3]{x^4} - \frac{2}{x} + 5x^4\right)' = \left(\frac{9}{x^3}\right)' + \left(\sqrt[3]{x^4}\right)' - \left(\frac{2}{x}\right)' + \left(5x^4\right)' =$$

$$= -\frac{27}{x^4} + \frac{4}{3}\sqrt[3]{x} + \frac{2}{x^2} + 20x^3$$

Example 28

$$y' = \left(\frac{\log_3(3x-7)}{ctg7x^5}\right)' = \frac{(\log_3(3x-7))' ctg7x^5 - \log_3(3x-7)(ctg7x^5)'}{ctg^2 7x^5} =$$

$$= \frac{\frac{(3x-7)'}{(3x-7)\ln 3} ctg7x^5 + \frac{(7x^5)'}{\sin^2 7x^5}\log_3(3x-7)}{ctg7x^5} =$$

$$= \frac{\frac{3}{(3x-7)\ln 3} ctg7x^5 + \frac{35x^4}{\sin^2 7x^5}\log_3(3x-7)}{ctg7x^5}$$

Example 29

$$y'(x) = \left(\ln x^2\right)' = \frac{1}{x^2} \cdot \left(x^2\right)' = \frac{1}{x^2} \cdot 2x = \frac{2x}{x^2} = \frac{2}{x} \quad (x \neq 0)$$

Example 30

$$y'(x) = \left(\ln^2 x\right)' = 2\ln x \cdot (\ln x)' = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x} \quad (x > 0)$$

Example 31

$$y'(x) = \left(\cos x^3\right)' = \sin x^3 \cdot \left(x^3\right)' = \sin x^3 \cdot 3x^2 = 3x^2 \sin x^3$$

Example 32

$$y'(x) = [\cos(3x+2)]' = -\sin(3x+2) \cdot (3x+2)' = -3\sin(3x+2)$$

Example 33

$$y'(x) = \left(\cos^4 x\right)' = 4\cos^3 x \cdot (\cos x)' = 4\cos^3 x \cdot (-\sin x) = -4\cos^3 x \sin x.$$

Example 34

$$y'(x) = \left(3^{\cos x}\right)' = 3^{\cos x} \cdot \ln 3 \cdot (\cos x)' = -3^{\cos x} \ln 3 \sin x$$

Example 35

$$y'(x) = (\ln \sin x)' = \frac{1}{\sin x} \cdot (\sin x)' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$$

Example 36

$$y'(x) = \left(\sqrt{x^2+2x+3}\right)' = \frac{1}{2\sqrt{x^2+2x+3}} \cdot \left(x^2+2x+3\right)' = \frac{2x+2}{2\sqrt{x^2+2x+3}}$$

$$= \frac{2(x+1)}{2\sqrt{x^2+2x+3}} = \frac{x+1}{\sqrt{x^2+2x+3}}.$$

Example 37

$$y'(x) = (\ln \ln x)' = \frac{1}{\ln x} \cdot (\ln x)' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

Example 38

$$y'(x) = \left[\left(\sqrt{x}-2\right)^7\right]' = 7\left(\sqrt{x}-2\right)^6 \cdot \left(\sqrt{x}-2\right)' = 7\left(\sqrt{x}-2\right)^6 \cdot \frac{1}{2\sqrt{x}} = \frac{7\left(\sqrt{x}-2\right)^6}{2\sqrt{x}} \quad (x > 0)$$

Example 39

$$y'(x) = \left(\log_3 x^2\right)' = \frac{1}{x^2 \ln 3} \cdot \left(x^2\right)' = \frac{2x}{x^2 \ln 3} = \frac{2}{x \ln 3} \quad (x \neq 0)$$

Example 40

$$y'(x) = \left(\sqrt{3x^2+1}\right)' = \frac{1}{2\sqrt{3x^2+1}} \cdot (3x^2+1)' = \frac{6x}{2\sqrt{3x^2+1}} = \frac{3x}{\sqrt{3x^2+1}}$$

Example 41

$$y'(x) = \left(\sin\frac{x}{3}\right)' = \cos\frac{x}{3} \cdot \left(\frac{x}{3}\right)' = \frac{1}{3}\cos\frac{x}{3}$$

Example 42

$$y'(x) = \left(\sqrt{\sin 2x+1}\right)' = \frac{1}{2\sqrt{\sin 2x+1}} \cdot (\sin 2x+1)' = \frac{1}{2\sqrt{\sin 2x+1}} \cdot \cos 2x \cdot (2x)'$$

$$= \frac{\not{2}\cos 2x}{\not{2}\sqrt{\sin 2x+1}} = \frac{\cos 2x}{\sqrt{\sin 2x+1}}.$$

Example 43

$$y'(x) = \left(x\sqrt{1+x^2}\right)' = x' \cdot \sqrt{1+x^2} + x \cdot \left(\sqrt{1+x^2}\right)' = 1 \cdot \sqrt{1+x^2} + x \cdot \frac{1}{2\sqrt{1+x^2}} \cdot (1+x^2)'$$

$$= \sqrt{1+x^2} + \frac{x}{2\sqrt{1+x^2}} \cdot 2x = \sqrt{1+x^2} + \frac{\not{2}x^2}{\not{2}\sqrt{1+x^2}} = \frac{\left(\sqrt{1+x^2}\right)^2 + x^2}{\sqrt{1+x^2}} = \frac{1+x^2+x^2}{\sqrt{1+x^2}}$$

$$= \frac{1+2x^2}{\sqrt{1+x^2}}.$$

Example 44

$$y'(x) = \left[\left(\frac{x+1}{x-1}\right)^3\right]' = 3\left(\frac{x+1}{x-1}\right)^2 \cdot \left(\frac{x+1}{x-1}\right)'$$

$$= 3\left(\frac{x+1}{x-1}\right)^2 \cdot \frac{(x+1)'(x-1)-(x+1)(x-1)'}{(x-1)^2} = 3\left(\frac{x+1}{x-1}\right)^2 \cdot \frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}$$

$$= 3\left(\frac{x+1}{x-1}\right)^2 \cdot \frac{\not{x}-1-\not{x}-1}{(x-1)^2} = 3\left(\frac{x+1}{x-1}\right)^2 \cdot \frac{(-2)}{(x-1)^2} = -6\frac{(x+1)^2}{(x-1)^4} \quad (x \neq 1).$$

Example 45

$$y'(x) = \left(\sqrt[3]{9x^2-1}\right)' = \frac{1}{3\sqrt[3]{(9x^2-1)^2}} \cdot (9x^2-1)' = \frac{18x}{3\sqrt[3]{(9x^2-1)^2}} = \frac{6x}{\sqrt[3]{(9x^2-1)^2}}$$

Example 46

$$y'(x) = \left[(5x+2)^{13} - (6x+7)^{10}\right]' = \left[(5x+2)^{13}\right]' - \left[(6x+7)^{10}\right]'$$

$$= 13(5x+2)^{12} \cdot (5x+2)' - 10(6x+7)^9 \cdot (6x+7)' = 13(5x+2)^{12} \cdot 5 - 10(6x+7)^9 \cdot 6$$

$$= 65(5x+2)^{12} - 60(6x+7)^9.$$

Example 47

$$y'(x) = \left[(x+\sqrt{x})^3\right]' = 3(x+\sqrt{x})^2 \cdot (x+\sqrt{x})' = 3(x+\sqrt{x})^2 \cdot \left(1+\frac{1}{2\sqrt{x}}\right)$$

$$= 3(x+\sqrt{x})^2 \cdot \frac{2\sqrt{x}+1}{2\sqrt{x}} = \frac{3(\sqrt{x})^2(\sqrt{x}+1)^2(2\sqrt{x}+1)}{2\sqrt{x}}$$

$$= \frac{3\sqrt{x}(\sqrt{x}+1)^2(2\sqrt{x}+1)}{2} \quad (x \geq 0).$$

Example 48

$$y'(x) = \left[\ln\left(\frac{x+1}{x-1}\right)\right]' = \frac{1}{\frac{x+1}{x-1}} \cdot \left(\frac{x+1}{x-1}\right)' = \frac{x-1}{x+1} \cdot \frac{(x+1)'(x-1)-(x+1)(x-1)'}{(x-1)^2}$$

$$= \frac{x-1}{x+1} \cdot \frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2} = \frac{x-1}{x+1} \cdot \frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2} = \frac{-2}{(x+1)(x-1)}.$$

Example 49

$$y'(x) = (\sin^4 x + \cos^4 x)' = (\sin^4 x)' + (\cos^4 x)' = 4\sin^3 x \cdot (\sin x)' + 4\cos^3 x \cdot (\cos x)'$$

$$= 4\sin^3 x \cos x - 4\cos^3 x \sin x = 4\sin x \cos x\,(\sin^2 x - \cos^2 x).$$

Example 50

$$y'(x) = (\log_5 \sin 2x)' = \frac{1}{\ln 5 \sin 2x} \cdot (\sin 2x)' = \frac{1}{\ln 5 \sin 2x} \cdot \cos 2x \cdot 2 = \frac{2\cos 2x}{\ln 5 \sin 2x} = \frac{2\cot 2x}{\ln 5}$$