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L I M I T S

OCTAVE

$\lim_{x\to 0} \frac{\sqrt{6-x}}{x^2-9}$	`sqrt(6-x)/(x^2-9)`
$\lim_{x\to\infty} \frac{\sqrt{6-x}}{\sqrt[3]{6+2x}}$	`sqrt(6-x)/(6+2*x)^(1/3)`
$\lim_{x\to\infty} \frac{\log_5(1-tan(x))}{\sin(x\cdot\pi)}$	`Log(1-tan(x),5)/sin(x*pi)`
$\lim_{x\to 1} \frac{x^2+2x-\frac{2}{3}}{x^3+x}$	`(x^2+2*x-2/3)/(x^3+x)`
$\lim_{x\to\infty} \left(\frac{3-3x}{4-3x}\right)^{2x+1}$	`((3-3*x)/(4-3*x))^(2*x+1)`

Example 1:

$$\lim_{x\to 1}\left(x^2-3x+5\right)=1-3+5=3.$$

Example 2:

$$\lim_{x\to\infty}\left(x^2-3x+5\right)=\lim_{x\to\infty}x^2\left(1-\left(\frac{3}{x}\right)^{\to 0}+\left(\frac{5}{x^2}\right)^{\to 0}\right)=\lim_{x\to\infty}x^2=\infty$$

Example 3:

$$\lim_{x\to 0}\frac{2x-1}{x^2+3x-4}=\lim_{x\to 0}\frac{2\cdot 0-1}{0^2+3\cdot 0-4}=\frac{1}{4}$$

Example 4:

$$\lim_{x\to 1}\frac{2x^2-3x-5}{x+1}=\lim_{x\to 1}\frac{2\cdot 1^2-3\cdot 1-5}{1+1}=-3$$

Example 5:

$$A=\lim_{x\to 2}\frac{x^2-6x+8}{x^2-8x+12}=$$

$$=\left\|\frac{2^2-6\cdot 2+8}{2^2-8\cdot 2+12}=\frac{0}{0}\right\|$$

$x^2 - 6x + 8 = 0;$
$D = 36 - 32 = 4;$
$x_1 = (6 + 2)/2 = 4;$
$x_2 = (6 - 2)/2 = 2;$

$$a\left(x-x_1\right)\left(x-x_2\right)=(x-4)(x-2)$$

$x^2 - 8x + 12 = 0;$
$D = 64 - 48 = 16;$
$x_1 = (8 + 4)/2 = 6;$
$x_2 = (8 - 4)/2 = 2;$

$$a\left(x-x_1\right)\left(x-x_2\right)=(x-6)(x-2)$$

$$A=\lim_{x\to 2}\frac{(x-2)(x-4)}{(x-2)(x-6)}=$$

$$=\lim_{x\to 2}\frac{x-4}{x-6}=\frac{2-4}{2-6}=\frac{-2}{-4}=\frac{1}{2}$$

Example 6.1: Calculate the limit:

$$\lim_{x\to 4}\frac{x^2-6x+8}{x^2-5x+4}=\left(\frac{0}{0}\right)=$$
$$=\lim_{x\to 4}\frac{(x-4)(x-2)}{(x-4)(x-1)}=\lim_{x\to 4}\frac{x-2}{x-1}=\frac{2}{3}$$

Example 6.2: Calculate the limit:

$$\lim_{x\to 10}\frac{x^2-13x+30}{x^2-100}=\left(\frac{0}{0}\right)=\lim_{x\to 10}\frac{(x-3)(x-10)}{(x-10)(x+10)}=\lim_{x\to 10}\frac{x-3}{x+10}=\frac{7}{20}$$

Example 7: Calculate the limit:

$$\lim_{x\to 2}\frac{x^2-4}{x^2-4x+4}=\frac{0}{0}=$$
$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)^2}=$$
$$=\lim_{x\to 2}\frac{x+2}{x-2}=\frac{2+2}{2-2}=\frac{4}{0}=\infty$$

Example 8: Calculate the limit:

$$\lim_{x\to 2}\frac{x^2-5x+6}{x^3-2x^2-x+2}=\left(\frac{0}{0}\right)=\lim_{x\to 2}\frac{(x-2)(x-3)}{(x-2)(x^2-1)}=\lim_{x\to 2}\frac{x-3}{x^2-1}=-\frac{1}{3}$$

Example 9: Calculate the limit:

$$\lim_{x\to 2}\frac{x^2-4}{x^2-x-2}=\left[\frac{0}{0}\right]=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(x+1)}=\lim_{x\to 2}\frac{x+2}{x+1}=\frac{4}{3}$$

Example 10: Calculate the limit:

$$\lim_{x\to 2}\frac{x^2-5x+6}{x^2-12x+20}=\left(\frac{0}{0}\right)=\lim_{x\to 2}\frac{(x-3)(x-2)}{(x-2)(x-10)}=\lim_{x\to 2}\frac{(x-3)}{(x-10)}=\frac{2-3}{2-10}=\frac{1}{8}$$

Example 11: Calculate the limit:

$$A=\lim_{x\to\infty}\frac{3x^3+4x^2+7}{2x^3-3x^2+11}=\left\|\frac{\infty}{\infty}\right\|$$

We divide the numerator and the denominator by x to the greatest power, i.e. x^3.

$$A=\lim_{x\to\infty}\frac{\frac{3x^3}{x^3}+\frac{4x^2}{x^3}+\frac{7}{x^3}}{\frac{2x^3}{x^3}-\frac{3x^2}{x^3}+\frac{11}{x^3}}==\lim_{x\to\infty}\frac{3+\frac{4}{x}+\frac{7}{x^3}}{2-\frac{3}{x}+\frac{11}{x^3}}=\frac{3+0+0}{2-0+0}=\frac{3}{2}$$

Example 12.1: Calculate the limit:

$$\lim_{x\to\infty}\frac{(x+10)^2}{8x^2+10}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{x^2+20x+100}{8x^2+10}=\lim_{x\to\infty}\frac{1+\frac{20}{x}+\frac{100}{x^2}}{8+\frac{10}{x^2}}=\frac{1}{8}$$

Example 12.2: Calculate the limit:

$$\lim_{x\to\infty}\frac{7x^3+15x^2+9x+1}{5x^4+6x^2-3x-4}=\frac{\infty}{\infty}=$$

$$=\lim_{x\to\infty}\frac{\frac{7x^3+15x^2+9x+1}{x^4}}{\frac{5x^4+6x^2-3x-4}{x^4}}=\lim_{x\to\infty}\frac{\frac{7}{x}^{\to 0}+\frac{15}{x^2}^{\to 0}+\frac{9}{x^3}^{\to 0}+\frac{1}{x^4}^{\to 0}}{5+\frac{6}{x^2}^{\to 0}-\frac{3}{x^3}^{\to 0}-\frac{4}{x^4}^{\to 0}}=$$

$$=\frac{0+0+0+0}{5+0-0-0}=\frac{0}{5}=0$$

Example 13:

$$\lim_{x\to\infty}\frac{x^2+2x-3}{x^3+7x+1}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^2}-\frac{3}{x^3}}{1+\frac{7}{x^2}+\frac{1}{x^3}}=\frac{0}{1}=0$$

Example 14.1: Calculate the limit:

$$\lim_{x\to\infty}\frac{2x^2-3x-5}{x+1}=\frac{\infty}{\infty}=$$

$$=\lim_{x\to\infty}\frac{\frac{2x^2-3x-5}{x^2}}{\frac{x+1}{x^2}}=\lim_{x\to\infty}\frac{2-\frac{3}{x}^{\to 0}-\frac{5}{x^2}^{\to 0}}{\frac{1}{x}^{\to 0}+\frac{1}{x^2}^{\to 0}}=\frac{2}{0}=\infty$$

Example 14.2: Calculate the limit:

$$\lim_{x\to\infty}\frac{3x^2-2x+1}{\sqrt{25x^4-x+2}}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{3-\frac{2}{x}+\frac{1}{x^2}}{\sqrt{25-\frac{1}{x^3}+\frac{2}{x^4}}}=\frac{3}{\sqrt{25}}=\frac{3}{5}$$

Example 15:

$$A=\lim_{x\to 8}\frac{\sqrt{9+2x}-5}{x^2-6x-16}=\left\|\frac{0}{0}\right\|$$

The conjugate factor is $a+b=\sqrt{9+2x}+5$.

$$A=\lim_{x\to 8}\frac{\left(\sqrt{9+2x}-5\right)\left(\sqrt{9+2x}+5\right)}{\left(x^2-6x-16\right)\left(\sqrt{9+2x}+5\right)}=$$

$$=\lim_{x\to 8}\frac{\left(\sqrt{9+2x}\right)^2-5^2}{\left(x^2-6x-16\right)\left(\sqrt{9+2x}+5\right)}=$$

$$=\lim_{x\to 8}\frac{9+2x-25}{\left(x^2-6x-16\right)\left(\sqrt{9+2x}+5\right)}=$$

$$=\lim_{x\to 8}\frac{2(x-8)}{(x-8)(x+2)\left(\sqrt{9+2x}+5\right)}=$$

$$=\lim_{x\to 8}\frac{2}{(x+2)\left(\sqrt{9+2x}+5\right)}=\frac{1}{10}\cdot\frac{2}{8+2}=\frac{1}{50}$$

Example 16:

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(\sqrt{1+x+x^2}-1)(\sqrt{1+x+x^2}+1)}{x\cdot(\sqrt{1+x+x^2}+1)}=$$

$$=\lim_{x\to 0}\frac{1+x+x^2-1}{x\cdot(\sqrt{1+x+x^2}+1)}=\lim_{x\to 0}\frac{x(1+x)}{x\cdot(\sqrt{1+x+x^2}+1)}=\lim_{x\to 0}\frac{1+x}{(\sqrt{1+x+x^2}+1)}=\frac{1}{2}$$

Example 17.1:

$$\lim_{x\to 6}\frac{\sqrt{x-2}-2}{x-6}=\left[\frac{0}{0}\right]=\lim_{x\to 6}\frac{(\sqrt{x-2}-2)(\sqrt{x-2}+2)}{(x-6)(\sqrt{x-2}+2)}=$$

$$=\lim_{x\to 6}\frac{x-2-4}{(x-6)(\sqrt{x-2}+2)}=\lim_{x\to 6}\frac{1}{\sqrt{x-2}+2}=\frac{1}{4}$$

Example 17.2:

$$\lim_{x\to 2}\frac{\sqrt{3x-2}-2}{\sqrt{2x+5}-3}=\left(\frac{0}{0}\right)=\lim_{x\to 2}\frac{\left(\sqrt{3x-2}-2\right)\left(\sqrt{3x-2}+2\right)}{\left(\sqrt{2x+5}-3\right)\left(\sqrt{3x-2}+2\right)}=$$

$$=\lim_{x\to 2}\frac{3x-2-4}{\left(\sqrt{2x+5}-3\right)\left(\sqrt{3x-2}+2\right)}=\lim_{x\to 2}\frac{3(x-2)\left(\sqrt{2x+5}+3\right)}{\left(\sqrt{2x+5}-3\right)\left(\sqrt{2x+5}+3\right)\left(\sqrt{3x-2}+2\right)}=$$

$$=\lim_{x\to 2}\frac{3(x-2)\left(\sqrt{2x+5}+3\right)}{(2x+5-9)\left(\sqrt{3x-2}+2\right)}=\lim_{x\to 2}\frac{3(x-2)\left(\sqrt{2x+5}+3\right)}{2(x-2)\left(\sqrt{3x-2}+2\right)}=$$

$$=\frac{3}{2}\lim_{x\to 2}\frac{\sqrt{2x+5}+3}{\sqrt{3x-2}+2}=\frac{3}{2}\cdot\frac{3+3}{2+2}=\frac{18}{8}=\frac{9}{4}$$

Example 18:

$$\lim_{x\to 0}\frac{\sin 5x}{x}=\left\|\frac{0}{0}\right\|=\lim_{x\to 0}\underbrace{\frac{\sin 5x}{5x}}_{\to 1}\cdot 5=5$$

Example 19:

$$\lim_{x\to 0}\frac{\sin 5x}{tg\ 8x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{5x}{8x}=\frac{5}{8}$$

$$x\to 0,\ \sin 5x\ \sim 5x,\ tg 8x\sim 8x,$$

Example 20:

$$A=\lim_{x\to\infty}\left(\frac{x+2}{x-4}\right)^{x}=\left\|1^{\infty}\right\|,\text{ because}$$

$$\lim_{x\to\infty}\frac{x+2}{x-4}=\left\|\frac{\infty}{\infty}\right\|=\lim_{x\to\infty}\frac{\frac{x}{x}+\frac{2}{x}}{\frac{x}{x}-\frac{4}{x}}=\lim_{x\to\infty}\frac{1+\frac{2}{x}}{1-\frac{4}{x}}=\frac{1+0}{1-0}=1$$

Then

$$A=\lim_{x\to\infty}\left(1+\frac{x+2}{x-4}-1\right)^{x}=\lim_{x\to\infty}\left(1+\frac{6}{x-4}\right)^{x}=$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-4}{6}}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x-4}{6}}\right)^{\frac{x-4}{6}\cdot\frac{6}{x-4}x}=$$

$$= \lim_{x\to\infty}\left(\underbrace{\left(1+\frac{1}{\frac{x-4}{6}}\right)^{\frac{x-4}{6}}}_{\to e}\right)^{\frac{6}{x-4}\cdot x} =$$

$$= \lim_{x\to\infty} e^{\frac{6}{x-4}\cdot x} = e^{\lim\limits_{x\to\infty}\frac{6x}{x-4}} =$$

$$= e^{\lim\limits_{x\to\infty}\frac{6x}{x-4}} = \left\|\frac{\infty}{\infty}\right\| = e^{\lim\limits_{x\to\infty}\frac{6x/x}{\frac{x}{x}-\frac{4}{x}}} =$$

$$= e^{\lim\limits_{x\to\infty}\frac{6}{1-\frac{4}{x}}} = e^{\frac{6}{1-0}} = e^{6}$$

Example 21:

$$\lim_{x\to\infty}\left(\frac{3x+1}{3x-5}\right)^{4x+7} = |1^{\infty}| = \lim_{x\to\infty}\left(1+\frac{3x+1}{3x-5}-1\right)^{4x+7} = \lim_{x\to\infty}\left(1+\frac{6}{3x-5}\right)^{4x+7} =$$

$$= \lim_{x\to\infty}\left(1+\frac{1}{\frac{3x-5}{6}}\right)^{4x+7} = \lim_{x\to\infty}\left(1+\frac{1}{\frac{3x-5}{6}}\right)^{\frac{3x-5}{6}\cdot\frac{6}{3x-5}\cdot(4x+7)} = \lim_{x\to\infty}\left(\left(1+\frac{1}{\frac{3x-5}{6}}\right)^{\frac{3x-5}{6}}\right)^{\frac{6\cdot(4x+7)}{3x-5}} = e^{8}$$

Example 22:

$$\lim_{x\to\infty}\left(\frac{2x^2+3}{2x^2-4}\right)^{3x}=|1^{\infty}|=\lim_{x\to\infty}\left(1+\frac{2x^2+3}{2x^2-4}-1\right)^{3x}=$$

$$=\lim_{x\to\infty}\left(1+\frac{7}{2x^2-4}\right)^{3x}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{2x^2-4}{7}}\right)^{3x}=$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{2x^2-4}{7}}\right)^{\frac{2x^2-4}{7}\cdot\frac{7}{2x^2-4}\cdot 3x}=\lim_{x\to\infty}\left(\left(1+\frac{1}{\frac{2x^2-4}{7}}\right)^{\frac{2x^2-4}{7}}\right)^{\frac{21x}{2x^2-4}}=e^0=1$$

Example 23:

$$\lim_{x\to 1}\left(7-6x\right)^{\frac{x}{3x-3}}=\left|\begin{matrix}y=x-1;\ x=y+1\\ y\to 0\end{matrix}\right|=\lim_{y\to 0}\left(7-6\cdot(y+1)\right)^{\frac{y+1}{3\cdot(y+1)-3}}=$$

$$=\lim_{y\to 0}\left(1-6y\right)^{\frac{y+1}{3y}}=\lim_{y\to 0}\left(1+(-6y)\right)^{\frac{1}{-6y}\cdot(-6y)\cdot\frac{y+1}{3y}}=\lim_{y\to 0}\left(\left(1+(-6y)\right)^{\frac{1}{-6y}}\right)^{-2(y+1)}=e^{-2}=\frac{1}{e^2}$$

Example 24:

$$\lim_{x\to 0}\frac{arctg2x}{8x}=\left\|\frac{0}{0}\right\|=$$

$$=\lim_{x\to 0}\frac{2x}{8x}=\frac{2}{8}=\frac{1}{4},$$

because $arctg\,2x\sim 2x$

Example 25:

$$\lim_{x\to 0}\frac{e^{3x^2}-1}{5x^2}=\left\|\frac{0}{0}\right\|=\lim_{x\to 0}\frac{3x^2}{5x^2}=\frac{3}{5},$$

because $e^{3x^2}-1\sim 3x^2$

Example 26:

$$\lim_{x\to\infty}\left(\sqrt{x^2+3x-2}-x\right)=\left\|\infty-\infty\right\|=$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+3x-2}-x\right)\left(\sqrt{x^2+3x-2}+x\right)}{\left(\sqrt{x^2+3x-2}+x\right)}=$$

$$=\lim_{x\to\infty}\frac{x^2+3x-2-x^2}{\sqrt{x^2+3x-2}+x}=$$

$$=\lim_{x\to\infty}\frac{3x-2}{\sqrt{x^2+3x-2}+x}=\left\|\frac{\infty}{\infty}\right\|=$$

$$=\lim_{x\to\infty}\frac{\frac{3x}{x}-\frac{2}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{3x}{x^2}-\frac{2}{x^2}}+\frac{x}{x}}=$$

$$=\lim_{x\to\infty}\frac{3-\frac{2}{x}}{\sqrt{1+\frac{3}{x}-\frac{2}{x^2}}+1}=$$

$$=\frac{3-0}{\sqrt{1+0-0}+1}=\frac{3}{1+1}=\frac{3}{2}$$

Example 27:

$$\lim_{x\to\infty}(x-\sqrt{x^2+5x-8})=(\infty-\infty)=\lim_{x\to\infty}\frac{(x-\sqrt{x^2+5x-8})(x+\sqrt{x^2+5x-8})}{x+\sqrt{x^2+5x-8}}=$$

$$=\lim_{x\to\infty}\frac{x^2-x^2-5x+8}{x+\sqrt{x^2+5x-8}}=\lim_{x\to\infty}\frac{8-5x}{x+\sqrt{x^2+5x-8}}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{8/x-5}{1+\sqrt{1+\frac{5}{x}-\frac{8}{x^2}}}=-\frac{5}{2}$$

Example 28:

$$\lim_{x\to2}\left(\frac{1}{x-2}-\frac{3}{x^2-4}\right)=\lim_{x\to2}\left(\frac{x^2-3x+2}{x^2-4}\right)=\left(\frac{0}{0}\right)=$$

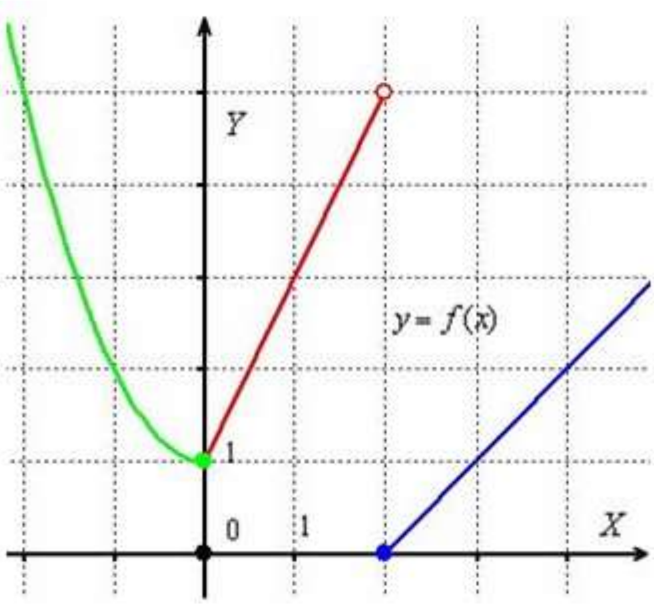
$$=\lim_{x\to2}\frac{(x-1)(x-2)}{(x-2)(x+2)}=\lim_{x\to2}\left(\frac{x-1}{x+2}\right)=\frac{1}{4}$$

Example. Investigate a continuity of the function: $f(x)=\begin{cases} x^2+1, & x\le 0 \\ 1+2x, & 0<x<2 \\ x-2, & x\ge 2 \end{cases}$

Solution.

We have 2 break points: $x=0 \quad and \quad x=2$

Let's investigate the point $x=2$.



The value of the function:

$f(2) = 2-2 = 0$

The left limit:

$\lim_{x\to 2-0}(1+2x) = 1+2\cdot 2 = 5$

The right limit:

$\lim_{x\to 2+0}(x-2) = 2-2 = 0$

Since

$\lim_{x\to 2-0}(1+2x) \neq \lim_{x\to 2+0}(x-2)$

we have the discontinuity of the first kind.