# TECHNIQUES OF CALCULATIONS OF LIMITS. TASKS

## Calculate limits of the following functions:

| Task 1 | Task 2 | Task 3 | Task 4 |
|---|---|---|---|
| $\lim\limits_{x\to 5} x^2$ | $\lim\limits_{x\to \frac{1}{2}} \dfrac{1}{x}$ | $\lim\limits_{x\to 0} \dfrac{x^2-5}{3x^2+x-10}$ | $\lim\limits_{x\to \frac{\pi}{2}} \sin x$ |
| Task 5 | Task 6 | Task 7 | Task 8 |
| $\lim\limits_{x\to 3} \dfrac{1+x}{2x^2-3x-5}$ | $\lim\limits_{x\to -3} \sqrt{x+4}$ | $\lim\limits_{x\to 2} 5^x$ | $\lim\limits_{x\to 8} \log_2 x$ |

### Indeterminate form $\dfrac{0}{0}$

| Task 9 | Task 10 | Task 11 | Task 12 |
|---|---|---|---|
| $\lim\limits_{x\to 1} \dfrac{x^2-1}{3x^2-2x-1}$ | $\lim\limits_{x\to 4} \dfrac{x^2-3x-4}{x^2-16}$ | $\lim\limits_{x\to 3} \dfrac{x^2-3x}{x^2-6x+9}$ | $\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{x-1}$ |
| Task 13 | Task 14 | Task 15 | Task 16 |
| $\lim\limits_{x\to 1} \dfrac{3x^2-2x-1}{x^4-x}$ | $\lim\limits_{x\to -1} \dfrac{3x^2+2x-1}{-x^2+x+2}$ | $\lim\limits_{x\to \frac{1}{3}} \dfrac{3x^2+2x-1}{27x^3-1}$ | $\lim\limits_{x\to -2} \dfrac{x^3+3x^2+2x}{x^2-x-6}$ |
| Task 17 | Task 18 | Task 19 | Task 20 |
| $\lim\limits_{x\to 1} \dfrac{x^3-x^2-x+1}{x^3+x^2-x-1}$ | $\lim\limits_{x\to 1} \dfrac{x^3+x-2}{x^3-x^2-x+1}$ | $\lim\limits_{x\to 3} \dfrac{x^2-9}{x^2+7x+12}$ | $\lim\limits_{x\to 4} \dfrac{x^2-6x+8}{x^2-5x+4}$ |
| Task 21 | Task 22 | Task 23 | Task 24 |
| $\lim\limits_{x\to 7} \dfrac{x^2-8x+7}{x^2-9x+14}$ | $\lim\limits_{x\to 2} \dfrac{x^3-8}{x^2-5x+6}$ | $\lim\limits_{x\to 3} \dfrac{x^2-6x+9}{2x^2-18}$ | $\lim\limits_{x\to -3} \dfrac{x^3+27}{x^2-9}$ |

### Indeterminate form $\dfrac{\infty}{\infty}$

| Task 25 | Task 26 | Task 27 | Task 28 |
|---|---|---|---|
| $\lim\limits_{x\to \infty} \dfrac{3x+1}{x}$ | $\lim\limits_{x\to \infty} \dfrac{3x^2-2x-1}{x^2+3x+7}$ | $\lim\limits_{x\to \infty} \dfrac{x^4-2x-1}{100x^3+x^2+2}$ | $\lim\limits_{x\to \infty} \dfrac{3x^2+5}{7x^4-2x^2+3}$ |
| Task 29 | Task 30 | Task 31 | Task 32 |
| $\lim\limits_{x\to +\infty} \dfrac{3^x+2}{3^{x+1}-1}$ | $\lim\limits_{x\to \pm\infty} \dfrac{2\cdot 5^x-3}{9\cdot 5^x+4}$ | $\lim\limits_{x\to -\infty} \dfrac{4^x+3^{x+1}}{4^{x+1}+3^x}$ | $\lim\limits_{x\to \infty} \dfrac{2x^2-3x-4}{\sqrt{x^4+1}}$ |
| Task 33 | Task 34 | Task 35 | Task 36 |

| | | | |
|---|---|---|---|
| $\lim\limits_{x\to\infty}\dfrac{x}{\sqrt[3]{x^3+9}}$ | $\lim\limits_{x\to\infty}\dfrac{4x^5-3x+2}{3+2x^3+5x^5}$ | $\lim\limits_{x\to\infty}\dfrac{3x^3-x^2+2x}{x^3-x^4}$ | $\lim\limits_{x\to\infty}\dfrac{7x^4+2x^3-1}{3x^2-2x^4+x}$ |

| Task 37 | Task 38 | Task 39 | Task 40 |
|---|---|---|---|
| $\lim\limits_{x\to\infty}\dfrac{5x^2-x^3-15}{x^2-16}$ | $\lim\limits_{x\to\infty}\dfrac{3x^5+2x^7}{2x^5-8x^7}$ | $\lim\limits_{x\to+\infty}\dfrac{3^x+2^x}{1-3^x}$ | $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+1}+x}{4x+1}$ |

| Task 41 | Task 42 | Task 43 | Task 44 |
|---|---|---|---|
| $\lim\limits_{x\to\infty}\dfrac{2x-3x^2}{5x^3+9}$ | $\lim\limits_{x\to\infty}\dfrac{5x^3+3x^2}{2x^4+3x-5}$ | $\lim\limits_{x\to\infty}\dfrac{7x-2x^4}{4x^4+3x^2+1}$ | $\lim\limits_{x\to-\infty}\dfrac{4^x+5^x}{4^{x+1}-5^{x+1}}$ |

Limits with $\left\|\dfrac{0}{0}\right\|$, containing **irrational expressions**

| Task 45 | Task 46 | Task 47 | Task 48 |
|---|---|---|---|
| $\lim\limits_{x\to4}\dfrac{x-\sqrt{3x+4}}{16-x^2}$ | $\lim\limits_{x\to2}\dfrac{\sqrt{x-1}-1}{x-2}$ | $\lim\limits_{x\to1}\dfrac{\sqrt{x}-\sqrt{2-x}}{2x^2-x-1}$ | $\lim\limits_{x\to3}\dfrac{x^2-9}{\sqrt{x+1}-\sqrt{3x-5}}$ |

| Task 49 | Task 50 | Task 51 | Task 52 |
|---|---|---|---|
| $\lim\limits_{x\to2}\dfrac{\sqrt{4x+1}-3}{\sqrt{x+2}-2}\cdot$ | $\lim\limits_{x\to-2}\dfrac{2-\sqrt{6+x}}{\sqrt{7-x}-3}$ | $\lim\limits_{x\to4}\dfrac{2x^2-9x+4}{\sqrt{5-x}-\sqrt{x-3}}$ | $\lim\limits_{x\to-1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}$ |

$\infty-\infty$

| Task 53 | Task 54 | Task 55 | Task 56 |
|---|---|---|---|
| $\lim\limits_{x\to-1}\left(\dfrac{1}{x+1}-\dfrac{1}{x^2-1}\right)$ | $\lim\limits_{x\to0}\left(\dfrac{1}{2x^2-x}-\dfrac{1}{x^2-x}\right)$ | $\lim\limits_{x\to\infty}\left(x-\dfrac{x^3}{x^2+1}\right)$ | $\lim\limits_{x\to\infty}\left(\dfrac{x^4}{x^2-2}-\dfrac{x^4}{x^2+2}\right)$ |

| Task 57 | Task 58 | Task 59 | Task 60 |
|---|---|---|---|
| $\lim\limits_{x\to\infty}\left(\dfrac{4x^4}{x^2+x+2}-4x^2\right)$ | $\lim\limits_{x\to+\infty}\left(\sqrt{x-3}-\sqrt{x+2}\right)$ | $\lim\limits_{x\to\pm\infty}\left(\sqrt{x^2+10x}-x\right)$ | $\lim\limits_{x\to\pm\infty}\left(\sqrt{4x^2+3x}-2x\right)$ |

| Task 61 | Task 62 | Task 63 | Task 64 |
|---|---|---|---|
| $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$ | $\lim\limits_{x\to+\infty}\left(\sqrt{2x+3}-\sqrt{x+5}\right)$ | $\lim\limits_{x\to1}\left(\dfrac{1}{1-x}-\dfrac{3}{1-x^2}\right)$ | $\lim\limits_{x\to2}\left(\dfrac{1}{x-2}-\dfrac{12}{x^3-8}\right)$ |

| Task 65 | Task 66 | Task 67 | Task 68 |
|---|---|---|---|
| $\lim\limits_{x\to1}\left(\dfrac{1}{x^2-x}-\dfrac{3}{x^3-1}\right)$ | $\lim\limits_{x\to3}\left(\dfrac{1}{x-3}-\dfrac{6}{x^2-9}\right)$ | $\lim\limits_{x\to\infty}\left(\dfrac{3x^3}{5x-1}-7x\right)$ | $\lim\limits_{x\to\infty}\left(\dfrac{3x^4}{x^2+3}-3x^2\right)$ |

Doc. Misiura Ie.Iu.

**The first remarkable limit** $\left[\dfrac{0}{0}\right]$ **and consequences**

$$\lim_{x\to 0}\frac{\sin x}{x}=\left\|\frac{0}{0}\right\|=1,\qquad \lim_{x\to 0}\frac{\sin f(x)}{f(x)}=\left\|\frac{0}{0}\right\|=1,\qquad \lim_{x\to 0}\frac{\arcsin f(x)}{f(x)}=\left\|\frac{0}{0}\right\|=1$$

$$\lim_{x\to 0}\frac{tg\ f(x)}{f(x)}=\left\|\frac{0}{0}\right\|=1,\qquad \lim_{x\to 0}\frac{arctg\ f(x)}{f(x)}=\left\|\frac{0}{0}\right\|=1$$

**Use** $\sin\alpha\pm\sin\beta=2\sin\dfrac{\alpha\pm\beta}{2}\cdot\cos\dfrac{\alpha\mp\beta}{2};\quad \cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\cdot\sin\dfrac{\alpha-\beta}{2},$

$\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2};$

| Task 69 | Task 70 | Task 71 | Task 72 |
|---|---|---|---|
| $\displaystyle\lim_{x\to 0}\frac{\sin 2x}{x}$ | $\displaystyle\lim_{x\to 0}\frac{\sin^2 5x}{3x^2}$ | $\displaystyle\lim_{x\to 0}\frac{tg^3 4x}{10x^3}$ | $\displaystyle\lim_{x\to 0}\frac{\sin 3x}{8x^2}$ |
| Task 73 | Task 74 | Task 75 | Task 76 |
| $\displaystyle\lim_{x\to 0}\frac{\sin^2 6x}{2x}$ | $\displaystyle\lim_{x\to 0}\frac{6x^3}{\sin^3 2x}$ | $\displaystyle\lim_{x\to 0}\frac{arctg 8x}{7x}$ | $\displaystyle\lim_{x\to 0}\frac{4x}{\arcsin 9x}$ |
| Task 77 | Task 78 | Task 79 | Task 80 |
| $\displaystyle\lim_{x\to 0}\frac{tg^2\frac{x}{2}}{2x}$ | $\displaystyle\lim_{x\to 0}\frac{tg^2 2x}{tg^2 3x}$ | $\displaystyle\lim_{x\to 0}\frac{tg 5x}{\sin 3x}$ | $\displaystyle\lim_{x\to 0}\frac{\sin 7x}{\sin 3x}$ |
| Task 81 | Task 82 | Task 83 | Task 84 |
| $\displaystyle\lim_{x\to 0}3x\cdot ctg 2x$ | $\displaystyle\lim_{x\to 0}\frac{1-\cos 6x}{tg^2 6x}$ | $\displaystyle\lim_{x\to 0}\frac{tg 6x}{\sin 8x}$ | $\displaystyle\lim_{x\to 0}x^2 ctg 3x$ |
| Task 85 | Task 86 | Task 87 | Task 88 |
| $\displaystyle\lim_{x\to 0}\frac{\arcsin^3 3x}{\arcsin^3 4x}$ | $\displaystyle\lim_{x\to 0}\frac{\sin 2x+\sin 8x}{12x}$ | $\displaystyle\lim_{x\to 0}\frac{\cos 4x-\cos 2x}{\arcsin^2 2x}$ | $\displaystyle\lim_{x\to 0}\frac{tg 5x}{\sin^2 3x}$ |
| Task 89 | Task 90 | Task 91 | Task 92 |
| $\displaystyle\lim_{x\to 0}\frac{1-\cos 3x}{tg^2 6x}$ | $\displaystyle\lim_{x\to 0}\frac{\sin^3 2x}{3x^3}$ | $\displaystyle\lim_{x\to 0}\frac{\sin 6x+\sin 2x}{4x}$ | $\displaystyle\lim_{x\to 0}\frac{\cos 6x-\cos 2x}{2x\cdot\sin x}$ |
| Task 93 | Task 94 | Task 95 | Task 96 |
| $\displaystyle\lim_{x\to 0}\frac{arctg 10x}{\sin 5x}$ | $\displaystyle\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$ | $\displaystyle\lim_{x\to 0}\frac{\arcsin^3 2x}{\sin^3 3x}$ | $\displaystyle\lim_{x\to 0}\frac{\sin^2 5x}{arctg 10x}$ |

| Task 97 | Task 98 | Task 99 | Task 100 |
|---|---|---|---|
| $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{x\cdot\sin 2x}$ | $\lim\limits_{x\to 0}\dfrac{\sin 6x+\sin 2x}{4x}$ | $\lim\limits_{x\to 0}\dfrac{\sin 3x-\sin 7x}{\sin 5x}$ | $\lim\limits_{x\to 0}\dfrac{tg^4 x}{arctg x^3}$ |

| The second remarkable limit $\left[1^\infty\right]$ |
|---|
| $$\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\left\|1^\infty\right\|=e$$ |

| Task 101 | Task 102 | Task 103 | Task 104 |
|---|---|---|---|
| $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{8x}$ | $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{x}$ | $\lim\limits_{x\to\infty}\left(1+\dfrac{7}{x}\right)^{2x}$ | $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{\frac{x+1}{5}}$ |
| **Task 105** | **Task 106** | **Task 107** | **Task 108** |
| $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x+5}\right)^{x}$ | $\lim\limits_{x\to\infty}\left(\dfrac{x+1}{6x+5}\right)^{x}$ | $\lim\limits_{x\to\infty}\left(\dfrac{5x+3}{x-1}\right)^{x}$ | $\lim\limits_{x\to\infty}\left(\dfrac{x^2+2}{2x^2+1}\right)^{x^2}$ |
| **Task 109** | **Task 110** | **Task 111** | **Task 112** |
| $\lim\limits_{x\to\infty}\left(\dfrac{3x+2}{3x-1}\right)^{\frac{x-2}{3}}$ | $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{3x+5}\right)^{2x}$ | $\lim\limits_{x\to+\infty}\left(\dfrac{5x-2}{2x+1}\right)^{5x}$ | $\lim\limits_{x\to+\infty}\left(\dfrac{5x^2+3}{10x^2-1}\right)^{\frac{x}{2}}$ |
| **Task 113** | **Task 114** | **Task 115** | **Task 116** |
| $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{2x-1}\right)^{x+1}$ | $\lim\limits_{x\to\infty}\left(\dfrac{2x^2+1}{2x^2-2}\right)^{x^2}$ | $\lim\limits_{x\to\infty}\left(\dfrac{x}{x+1}\right)^{x}$ | $\lim\limits_{x\to\infty}\left(\dfrac{4x-1}{3x+2}\right)^{5x}$ |
| **Task 117** | **Task 118** | **Task 119** | **Task 120** |
| $\lim\limits_{x\to\infty}\left(1-\dfrac{1}{x-1}\right)^{2x}$ | $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+1}\right)^{x}$ | $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x-1}\right)^{2x}$ | $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x+3}\right)^{x}$ |

## Calculate limits, using equivalencies

| | |
|---|---|
| $\sin f(x) \sim f(x)$ <br> $\arcsin f(x) \sim f(x)$ <br> $tg\ f(x) \sim f(x)$ <br> $arctg\ f(x) \sim f(x)$ <br> $1 - \cos f(x) \sim \dfrac{1}{2} f^2(x)$ | $\log_a(1 + f(x)) \sim \dfrac{f(x)}{\ln a}$ <br> $\ln(1 + f(x)) \sim f(x)$ <br> $a^{f(x)} - 1 \sim f(x) \cdot \ln a$, if $a > 0, a \neq 1$ <br> $e^{f(x)} - 1 \sim f(x)$, <br> $(1 + x)^m - 1 \sim m \cdot x$ |

| Task 121 | Task 122 | Task 123 | Task 124 |
|---|---|---|---|
| $\lim\limits_{x \to 0} \dfrac{e^{3x} - 1}{\ln(1 - 9x)}$ | $\lim\limits_{x \to 0} \dfrac{e^{3x} - 1}{4x}$ | $\lim\limits_{x \to 0} \dfrac{\sin 20x}{tg 15x}$ | $\lim\limits_{x \to 0} \dfrac{1 - \cos 6x}{\sin^2 4x}$ |
| **Task 125** | **Task 126** | **Task 127** | **Task 128** |
| $\lim\limits_{x \to 0} \dfrac{arctg\left(\dfrac{x}{2}\right)}{\sin 4x}$ | $\lim\limits_{x \to 0} \dfrac{\arcsin^3 5x}{x^2 tg 5x}$ | $\lim\limits_{x \to 0} \dfrac{\sin 3x}{3x + x^2}$ | $\lim\limits_{x \to 0} \dfrac{\sin^2(3x^3)}{3x^2 arctg^2(2x^2)}$ |
| **Task 129** | **Task 130** | **Task 131** | **Task 132** |
| $\lim\limits_{x \to 0} \dfrac{x^2 tg 3x}{\arcsin \dfrac{x^3}{3}}$ | $\lim\limits_{x \to 0} \dfrac{\sin^2 3x}{\ln^2(1 + 2x)}$ | $\lim\limits_{x \to 0} \dfrac{arctg 5x}{e^{-3x} - 1}$ | $\lim\limits_{x \to 0} \dfrac{\sin 3x \sin 5x}{e^{2x^2} - 1}$ |
| **Task 133** | **Task 134** | **Task 135** | **Task 136** |
| $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + 2x} - 1}{tg 3x}$ | $\lim\limits_{x \to 0} \dfrac{1 - \cos 5x}{\sin^2 8x}$ | $\lim\limits_{x \to 0} \dfrac{arctg^3 4x}{8x^3}$ | $\lim\limits_{x \to 0} \dfrac{\arcsin\left(\dfrac{5x}{6}\right)}{arctg 2x}$ |