Example (Variant 30)

Task 1. Coordinates of vertices of the pyramid $ABCD$ are given as: $A(-7,-5,6)$; $B(-2,5,-3)$; $C(3,-2,4)$; $D(1,2,2)$.

Find:

1) lengths of edges DA, DB, DC as modules of vectors $\overrightarrow{DA}, \overrightarrow{DB}, \overrightarrow{DC}$;

$$\overrightarrow{DA} = (-7-1;-5-2;6-2) = (-8;-7;4)$$

$$\overrightarrow{DB} = (-2-1;5-2;-3-2) = (-3;3;-5)$$

$$\overrightarrow{DC} = (3-1;-2-2;4-2) = (2;-4;2)$$

USE OCTAVE

https://octave-online.net/,

```
octave:1> pA=[-7 -5 6]
pA =

  -7  -5   6

octave:2> pB=[-2 5 -3]
pB =

  -2   5  -3

octave:3> pC=[3 -2 4]
pC =

   3  -2   4

octave:4> pD=[1 2 2]
pD =

   1   2   2
octave:5> vDA=pA-pD
vDA =

  -8  -7   4

octave:6> vDB=pB-pD
vDB =

  -3   3  -5

octave:7> vDC=pC-pD
vDC =

   2  -4   2
```

$|\overrightarrow{DA}| = \sqrt{(-8)^2 + (-7)^2 + 4^2} = \sqrt{64 + 49 + 16} = \sqrt{129} \approx 11{,}358$ (units of length)

$|\overrightarrow{DB}| = \sqrt{(-3)^2 + 3^2 + (-5)^2} = \sqrt{9 + 9 + 25} = \sqrt{43} \approx 6{,}557$ (units of length)

$|\overrightarrow{DC}| = \sqrt{2^2 + (-4)^2 + 2)^2} = \sqrt{4 + 16 + 4} = \sqrt{24} \approx 4{,}899$ (units of length)

USE OCTAVE
https://octave-online.net/,

```
octave:8> lDA=norm(vDA)
lDA = 11.358
octave:9> lDB=norm(vDB)
lDB = 6.5574
octave:10> lDC=norm(vDC)
lDC = 4.8990
```

2) the angle D between edges DA and DB as the angle between two vectors $\vec{a} = \overrightarrow{DA}$ and $\vec{b} = \overrightarrow{DB}$;

$\overrightarrow{DA} = (-7 - 1; -5 - 2; 6 - 2) = (-8; -7; 4)$

$\overrightarrow{DB} = (-2 - 1; 5 - 2; -3 - 2) = (-3; 3; -5)$

$|\overrightarrow{DA}| = \sqrt{(-8)^2 + (-7)^2 + 4^2} = \sqrt{64 + 49 + 16} = \sqrt{129} \approx 11{,}358$ (units of length)

$|\overrightarrow{DB}| = \sqrt{(-3)^2 + 3^2 + (-5)^2} = \sqrt{9 + 9 + 25} = \sqrt{43} \approx 6{,}557$ (units of length)

$\overrightarrow{DA} \cdot \overrightarrow{DB} = (-8) \cdot (-3) + (-7) \cdot 3 + 4 \cdot (-5) = 24 - 21 - 20 = -17$

$$\cos D = \frac{\overrightarrow{DA} \cdot \overrightarrow{DB}}{|\overrightarrow{DA}| \cdot |\overrightarrow{DB}|} = \frac{-17}{\sqrt{129} \cdot \sqrt{43}} \approx -0{,}2283$$

USE OCTAVE
https://octave-online.net/,

```
octave:11> scprDADB=dot(vDA,vDB)
scprDADB = -17
octave:12> cosD=scprDADB/(lDA*lDB)
cosD = -0.2283
```

3) the cross product of $\vec{a}=\overrightarrow{DA}$ and $\vec{b}=\overrightarrow{DB}$, i.e. $\vec{c}=\vec{a}\times\vec{b}=\overrightarrow{DA}\times\overrightarrow{DB}$;

$$\vec{c}=\overrightarrow{DA}\times\overrightarrow{DB}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -8 & -7 & 4\\ -3 & 3 & -5\end{vmatrix}\begin{matrix}\vec{i} & \vec{j}\\ -8 & -7\\ -3 & 3\end{matrix}=35\vec{i}-12\vec{j}-24\vec{k}-21\vec{k}-12\vec{i}-40\vec{j}=$$

$$=23\vec{i}-52\vec{j}-45\vec{k}$$

USE OCTAVE

https://octave-online.net/,

```
octave:13> c=cross(vDA,vDB)
c =

   23  -52  -45
```

4) the area of ABD as $S=\frac{1}{2}|\vec{c}|=\frac{1}{2}\sqrt{c_x^2+c_y^2+c_z^2}$;

$$|\vec{c}|=\sqrt{23^2+(-52)^2+(-45)^2}=\sqrt{5258}\approx 72{,}512$$

$$S_{ABD}=\frac{1}{2}|\vec{c}|=\frac{1}{2}\cdot 72{,}512\approx 36{,}256 \quad \text{(square units)}$$

USE OCTAVE

https://octave-online.net/,

```
octave:14> lc=norm(c)
lc = 72.512
octave:15> S_ABD=1/2*lc
S_ABD = 36.256
```

5) the mixed product of three vectors $\overrightarrow{DA},\overrightarrow{DB},\overrightarrow{DC}$;

$$\overrightarrow{DA}=(-7-1;-5-2;6-2)=(-8;-7;4)$$

$$\overrightarrow{DB}=(-2-1;5-2;-3-2)=(-3;3;-5)$$

$$\overrightarrow{DC}=(3-1;-2-2;4-2)=(2;-4;2)$$

$$\left(\overrightarrow{DA},\overrightarrow{DB},\overrightarrow{DC}\right)=\begin{vmatrix}-8 & -7 & 4\\ -3 & 3 & -5\\ 2 & -4 & 2\end{vmatrix}\begin{matrix}-8 & -7\\ -3 & 3\\ 2 & -4\end{matrix}=-48+70+48-24+160-42=164$$

USE OCTAVE

https://octave-online.net/,

```
octave:16> DADBDC=[vDA;vDB;vDC]
DADBDC =

  -8  -7   4
  -3   3  -5
   2  -4   2

octave:17> mixedprDADBDC=det(DADBDC)
mixedprDADBDC = 164
```

6) the volume of the pyramid $ABCD$ as $V=\frac{1}{6}\left|\left(\overrightarrow{DA},\overrightarrow{DB},\overrightarrow{DC}\right)\right|$;

$$V_{ABCD}=\frac{1}{6}\left|\left(\overrightarrow{DA},\overrightarrow{DB},\overrightarrow{DC}\right)\right|=\frac{1}{6}\cdot|164|=\frac{164}{6}=\frac{82}{3}\approx 27{,}33 \quad \text{(cubed units)}$$

USE OCTAVE

https://octave-online.net/,

```
octave:18> V_ABCD=1/6*abs(mixedprDADBDC)
V_ABCD = 27.333
```

7) coordinates of M as the midpoint of side DA.

$A(-7,-5,6)$ and $D(1,2,2)$

$$x_M=\frac{x_D+x_A}{2}=\frac{-7+1}{2}=-3$$

$$y_M=\frac{y_D+y_A}{2}=\frac{-5+2}{2}=-1{,}5$$

$$z_M=\frac{z_D+z_A}{2}=\frac{6+2}{2}=4$$

USE OCTAVE

https://octave-online.net/,

```
octave:19> pointM=midPoint(pA,pD)
pointM =

  -3.0000  -1.5000   4.0000
```