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Application of systems of linear equations to Business and Economics

Applications of System of Linear Equations

The following examples can be used to illustrate the common methods of solving systems of linear equations that result from applied business and economic problems.

Illustration 6 – Mr. X invested a par of his investment in 10% bond A and a part in 15% bond B. His interest income during the first year is Rs 4,000. If he invests 20% more in 10% bond A and 10 % more in 15% bond B, his income during the second year increases by Rs 500. Find his initial investment and the new investment in bonds A and B using matrix method.

Solution –

Let initial investment be x in 10% bond A and y in 15% bond B. Then, according to given information, we have

$$0.10x+0.15y=4{,}000 \quad \text{or} \quad 2x+3y=80{,}000$$

$$0.12x+0.165y=4{,}500 \quad \text{or} \quad 8x+11y=3{,}00{,}000$$

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Expressing the above equations in matrix form, we obtain

$$\underset{\mathbf{A}}{\begin{bmatrix} 2 & 3 \\ 8 & 11 \end{bmatrix}} \underset{\mathbf{X}}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{\mathbf{B}}{\begin{bmatrix} 80{,}000 \\ 3{,}00{,}000 \end{bmatrix}}$$

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Since $|A| = -2 \neq 0$, A^{-1} exists and the solution can be given by:

$$X = A^{-1}B$$

$$= -\frac{1}{2}\begin{bmatrix} 11 & -3 \\ -8 & 2 \end{bmatrix}\begin{bmatrix} 80{,}000 \\ 3{,}00{,}000 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -20{,}000 \\ -40{,}000 \end{bmatrix}$$

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$$= \begin{bmatrix} 10{,}000 \\ 20{,}000 \end{bmatrix}$$

Hence x = Rs 10,000, y = Rs 20,000, and new investments would be Rs 12,000 and Rs 22,000 respectively.

Illustration *7* – An automobile company uses three types of steel s_1, s_2 and s_3 for producing three types of cars c_1, c_2 and c_3. The steel requirement (in tons) for each type of car is given below:

		Cars		
		c_1	c_2	c_3
	s_1	2	3	4
Steel	s_2	1	1	2
	s_3	3	2	1

Determine the number of cars of each type which can be produced using 29, 13 and 16 tons of steel of the three types respectively.

Solution –
Let x, y and z denote the number of cars that can be produced of each type. Then we have

$$2x+3y+4z = 29$$
$$x+y+2z = 13$$
$$3x+2y+z = 16$$

Solution –
Let x, y and z denote the number of cars that can be produced of each type. Then we have

$$2x+3y+4z=29$$
$$x+y+2z=13$$
$$3x+2y+z=16$$

The above information can be represented using the matrix method, as under.

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

The above equation can be solved using Gauss Jordan elimination method.
By applying the operation $R_1 \leftrightarrow R_2$ the given system is equivalent to

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 29 \\ 16 \end{bmatrix}$$

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Now applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$, the above system is equivalent to

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 3 \\ -23 \end{bmatrix}$$

Applying $R_3 \rightarrow R_3 + R_2$ the above system is equivalent to

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & -1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 3 \\ -23 \end{bmatrix}$$

Applying $R_3 \to R_3 + R_2$ the above system is equivalent to

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 3 \\ -20 \end{bmatrix}$$

$$x + y + 2z = 13 \qquad -(1)$$

$$y = 3 \qquad -(2)$$

$$-5z = -20 \qquad -(3)$$

Therefore, $z = 4$. Substituting $y = 3$ and $z = 4$ in (1), we get $x = 2$. Hence the solution is

$x = 2, y = 3$ and $z = 4$

Illustration 8 – A company produces three products everyday. Their total production on a certain day is 45 tons. It is found that the production of the third product exceeds the production of the first product by 8 tons while the total combined production of the first and third product is twice that of the second product. Determine the production level of each product using Cramer's rule.

Solution –

Let the production level of the three products be x, y and z respectively. Therefore, we will have the following equations

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Solution –

Let the production level of the three products be x, y and z respectively. Therefore, we will have the following equations

$$x + y + z = 45 \qquad -(1)$$

$$z = x + 8$$

$$i.e. \quad -x + 0y + z = 8 \qquad -(2)$$

$$x + z = 2y$$

$$i.e. \quad x - 2y + z = 0 \qquad -(3)$$

Solution –

Let the production level of the three products be x, y and z respectively. Therefore, we will have the following equations

$$x+y+z=45 \qquad -(1)$$

$$z=x+8$$

$$i.e. \quad -x+0y+z=8 \qquad -(2)$$

$$x+z=2y$$

$$i.e. \quad x-2y+z=0 \qquad -(3)$$

Therefore, we have, using (1), (2) and (3)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

Therefore, we have, using (1), (2) and (3)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

Which gives us

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 6$$

Since $\Delta \neq 0$, there is a unique solution.

$$\Delta_1 = \begin{vmatrix} 45 & 1 & 1 \\ 8 & 0 & 1 \\ 0 & -2 & 1 \end{vmatrix} = 66$$

Therefore, we have, using (1), (2) and (3)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

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$$\Delta_2 = \begin{vmatrix} 1 & 45 & 1 \\ -1 & 8 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 90$$

Therefore, we have, using (1), (2) and (3)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

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$$\Delta_3 = \begin{vmatrix} 1 & 1 & 45 \\ -1 & 0 & 8 \\ 1 & -2 & 0 \end{vmatrix} = 114$$

Which gives us

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Since $\Delta \neq 0$, there is a unique solution.

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$$\Delta_3 = \begin{vmatrix} 1 & 1 & 45 \\ -1 & 0 & 8 \\ 1 & -2 & 0 \end{vmatrix} = 114$$

Therefore,

$$x = \frac{66}{6} = 11$$

$$y = \frac{90}{6} = 15$$

$$z = \frac{114}{6} = 19$$

Hence, the production levels of the products are as follows:

First product - 11 tons
Second product - 15 tons
Third product - 19 tons