

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Applications of Matrices to Business and Economics

What is a Matrix?

But first, let's discuss how various situations in business and economics can be represented using matrices. This can be done using the following examples.

1. Annual productions of two branches selling three types of items may be represented as follows:

<i>Branch</i>	<i>Item A</i>	<i>Item B</i>	<i>Item C</i>
<i>I</i>	2000	2876	2314
<i>II</i>	7542	3214	2969

What is a Matrix?

2. Number of staff in the office can be represented as follows:

2	<i>Peon</i>
4	<i>Clerk</i>
3	<i>Typist</i>
1	<i>Head Clerk</i>
1	<i>Office Superintendent</i>

What is a Matrix?

3. The unit cost of transportation of an item from each of the three factories to each of the four warehouses can be represented as follows:

	Warehouse			
Factory	W_1	W_2	W_3	W_4
<i>I</i>	13	12	17	14
<i>II</i>	22	26	11	19
<i>III</i>	16	15	18	11

Applications of Matrix Addition and Subtraction

The applications of addition and subtraction of matrices can be illustrated through the following examples:

Illustration 1 - The quarterly sales of Jute, Cotton and Yarn for the year 2018 and 2020 are given below.

$$\mathbf{A} = \begin{array}{l} \text{Jute} \\ \text{Cotton} \\ \text{Yarn} \end{array} \begin{array}{c} \text{Year 2018} \\ \text{Q1} \quad \text{Q2} \quad \text{Q3} \quad \text{Q4} \\ \left[\begin{array}{cccc} 20 & 25 & 22 & 20 \\ 10 & 20 & 18 & 10 \\ 15 & 20 & 15 & 15 \end{array} \right] \end{array}$$

$$\mathbf{B} = \begin{array}{l} \text{Jute} \\ \text{Cotton} \\ \text{Yarn} \end{array} \begin{array}{c} \text{Year 2020} \\ \left[\begin{array}{cccc} 10 & 15 & 20 & 20 \\ 5 & 20 & 18 & 10 \\ 8 & 30 & 15 & 10 \end{array} \right] \end{array}$$

Solution –

The total sales of Jute, Cotton and Yarn will be obtained as under

$$\begin{aligned} \mathbf{A + B} &= \begin{bmatrix} 20 & 25 & 22 & 20 \\ 10 & 20 & 18 & 10 \\ 15 & 20 & 15 & 15 \end{bmatrix} + \begin{bmatrix} 10 & 15 & 20 & 20 \\ 5 & 20 & 18 & 10 \\ 8 & 30 & 15 & 10 \end{bmatrix} \\ &= \begin{bmatrix} 30 & 40 & 42 & 40 \\ 15 & 40 & 36 & 40 \\ 23 & 50 & 30 & 25 \end{bmatrix} \end{aligned}$$

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Illustration 2 – X Ltd has the following sales position of its products A and B at its two centers P and Q at the end of the year

$$Y = \begin{array}{c} A \\ B \end{array} \begin{array}{cc} P & Q \\ \left[\begin{array}{cc} 50 & 45 \\ 60 & 70 \end{array} \right] \end{array}$$

If the sales for the first three months is given as

$$Q = \begin{array}{c} A \\ B \end{array} \begin{array}{cc} P & Q \\ \left[\begin{array}{cc} 30 & 15 \\ 20 & 20 \end{array} \right] \end{array}$$

Find the sales position for the last nine months.

Solution –

Given are the sales positions for the whole year (Y) and for the first three months (Q).

Hence, sales position for the remaining nine months –

$$\begin{aligned} Y - Q &= \begin{bmatrix} 50 & 45 \\ 60 & 70 \end{bmatrix} - \begin{bmatrix} 30 & 15 \\ 20 & 20 \end{bmatrix} \\ &\quad \quad \quad \begin{matrix} P & Q \end{matrix} \\ &= \begin{matrix} A \\ B \end{matrix} \begin{bmatrix} 20 & 30 \\ 40 & 50 \end{bmatrix} \end{aligned}$$

Applications of Matrix Multiplication

Illustration 3 – Ram, Shyam and Mohan purchased biscuits of different brands P, Q and R. Ram purchased 10 packets of P, 7 packets of Q and 3 packets of R. Shyam purchased 4 packets of P, 8 packets of Q and 10 packets of R. Mohan purchased 4 packets of P, 7 packets of Q and 8 packets of R. If brand P costs Rs 4, Q costs Rs 5 and R costs Rs 6 each, then using matrix operation, find the amount of money spent by these persons individually.

Solution –

Let Q be the matrix denoting the quantity of each brand of biscuit bought by P, Q and R and let C be the matrix showing the cost of each brand of biscuit.

$$Q = \begin{matrix} & P & Q & R \\ \begin{matrix} Ram \\ Shyam \\ Mohan \end{matrix} & \begin{bmatrix} 10 & 7 & 3 \\ 4 & 8 & 10 \\ 4 & 7 & 8 \end{bmatrix} \end{matrix}_{3 \times 3}$$

$$C = \begin{matrix} P \\ Q \\ R \end{matrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}_{3 \times 1}$$

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Since number of columns of first matrix should be equal to the number of rows of the second matrix for multiplication to be possible, the above matrices shall be multiplied in the following order.

$$\mathbf{Q} \times \mathbf{C} = \begin{bmatrix} 10 & 7 & 3 \\ 4 & 8 & 10 \\ 4 & 7 & 8 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

$$\mathbf{Q} \times \mathbf{C} = \begin{bmatrix} 10 & 7 & 3 \\ 4 & 8 & 10 \\ 4 & 7 & 8 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \times 4 + 7 \times 5 + 3 \times 6 \\ 4 \times 4 + 8 \times 5 + 10 \times 6 \\ 4 \times 4 + 7 \times 5 + 3 \times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 40 + 35 + 18 \\ 16 + 40 + 16 \\ 16 + 35 + 48 \end{bmatrix} = \begin{bmatrix} 93 \\ 116 \\ 99 \end{bmatrix}$$

Amount spent by Ram, Shyam and Mohan is Rs 99, Rs 116 and Rs 99 respectively.

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Illustration 4- A firm produces three products A, B and C requiring the mix of three materials P, Q and R. The requirement (per unit) of each product for each material is as follows.

$$\mathbf{M} = \begin{matrix} & \begin{matrix} P & Q & R \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix} \end{matrix}$$

Using matrix notations, find

- (i). The total requirement of each material if the firm produces 100 units of each product.
- (ii). The per unit cost of production of each product if the per unit cost of materials P, Q and R is Rs 5, Rs 10 and Rs 5 respectively.
- (iii). The total cost of production if the firm produces 200 units of each product.

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Solution –

- (i). The total requirement of each material if the firm produces 100 units of each product can be calculated using the matrix multiplication given below.

$$\begin{array}{ccc} & & P \quad Q \quad R \\ \begin{array}{ccc} A & B & C \\ [100 & 100 & 100] \end{array} & \begin{array}{c} \left[\begin{array}{ccc} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{array} \right] \begin{array}{l} A \\ B \\ C \end{array} \\ & = & \begin{array}{ccc} P & Q & R \\ [800 & 900 & 800] \end{array} \end{array}$$

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Solution –

- (i). The total requirement of each material if the firm produces 100 units of each product can be calculated using the matrix multiplication given below.

$$\begin{array}{c}
 \begin{array}{ccc}
 & P & Q & R \\
 A & B & C & \\
 \begin{bmatrix} 100 & 100 & 100 \end{bmatrix} & \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} & = & \begin{matrix} P & Q & R \\ 800 & 900 & 800 \end{matrix}
 \end{array}
 \end{array}$$

- (ii). Let the per unit cost of materials P, Q and R be represented by the 3×1 matrix as under

$$\begin{array}{c}
 C \\
 = \\
 \begin{matrix} P \\ Q \\ R \end{matrix}
 \end{array}
 \begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}$$

With the help of matrix multiplication, the per unit cost of production of each product would be calculated as under

$$\begin{array}{c}
 AC \\
 = \\
 \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix}
 \begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}
 = \begin{matrix} A \\ B \\ C \end{matrix}
 \begin{bmatrix} 45 \\ 65 \\ 60 \end{bmatrix}
 \end{array}$$

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

- (iii). The total cost of production if the firm produces 200 units of each product would be given as

$$[200 \quad 200 \quad 200] \begin{bmatrix} 45 \\ 65 \\ 60 \end{bmatrix} = [34,000]$$

Hence, the total cost of production will be Rs. 34,000.

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

Illustration 5 - Mr. X went to a market to purchase 3 kg of sugar, 10 kg of wheat and 1 kg of salt. In a shop near to Mr. X's residence, these commodities are priced at Rs 20, Rs 10 and Rs 8 per kg whereas in the local market these commodities are priced at Rs 15, Rs 8 and Rs 6 per kg respectively. If the cost of traveling to local market is Rs 25, find the net savings of Mr. X, using matrix multiplication method.

Solution –

Let matrices Q and P represent quantity and price. Then,

$$\text{Quantity Matrix} = Q = \begin{bmatrix} 3 & 10 & 1 \end{bmatrix}$$

$$\text{Price Matrix} = P = \begin{matrix} & \begin{matrix} \text{Shop} & \text{Local Market} \end{matrix} \\ \begin{matrix} \text{Sugar} \\ \text{Wheat} \\ \text{Salt} \end{matrix} & \begin{bmatrix} 20 & 15 \\ 10 & 8 \\ 8 & 6 \end{bmatrix} \end{matrix}$$

PhD Misiura Ie.Iu. (доцент Місюра Є.Ю.)

$$\text{Therefore, Total Price} = Q \times P = \begin{bmatrix} 3 & 10 & 1 \end{bmatrix} \begin{bmatrix} 20 & 15 \\ 10 & 8 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 168 & 131 \end{bmatrix}$$

Now,

Cost of purchasing from shop = Rs 168 and

Cost of purchasing from local market = Rs 131 + Rs 25 (Cost of travel) = Rs 156

Hence, net savings to Mr. X from purchasing through Local Market
= 168 – 156 = Rs 12

36