	Properties of power functions	**EXAMPLES**	**DERIVATIVES**
1	$x^m \cdot x^n = x^{m+n}$	$x^5 \cdot x^7 = x^{5+7} = x^{12}$	$\left(x^{12}\right)' = 12x^{11}$
2	$\left(x^m\right)^n = x^{m \cdot n}$	$\left(x^3\right)^2 = x^{3 \cdot 2} = x^6$	$\left(x^6\right)' = 6x^5$
3	$\frac{1}{x^n} = x^{-n}$	$\frac{1}{2x^3} = \frac{1}{2}x^{-3}$, $\frac{1}{x} = x^{-1}$	$\left(\frac{1}{2}x^{-3}\right)' = \frac{1}{2} \cdot (-3)x^{-4} = -\frac{3}{2}x^{-4}$
4	$\sqrt[m]{x^n} = x^{\frac{n}{m}}$	$3\sqrt[4]{x^5} = 3x^{\frac{5}{4}}$, $\sqrt{x} = x^{\frac{1}{2}}$	$\left(3x^{\frac{5}{4}}\right)' = 3 \cdot \frac{5}{4} \cdot x^{\frac{5}{4}-1} = \frac{15}{4} \cdot x^{\frac{1}{4}}$
5	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^{10}}{x^2} = x^{10-2} = x^8$	$\left(x^8\right)' = 8x^7$
6	$\left(\frac{x}{5}\right)' = \frac{x'}{5} = \frac{1}{5}$		

EXAMPLES

Example 1. Find a derivative of a function: $y = 4x^6 + \frac{1}{2x^3} - 3\sqrt[4]{x^5} + \frac{2}{3}$.

Then

$$y' = \left(4x^6 + \frac{1}{2x^3} - 3\sqrt[4]{x^5} + \frac{2}{3}\right)' = \left| \ (u \pm v)' = u' \pm v' \right| =$$

$$= \left(4x^6\right)' + \left(\frac{1}{2}x^{-3}\right)' - \left(3x^{\frac{5}{4}}\right)' + \left(\frac{2}{3}\right)' = \left| \begin{array}{l} C' = 0, (Cu)' = Cu' \\ \left(x^n\right)' = nx^{n-1} \end{array} \right| =$$

$$= 6 \cdot 4x^5 + \frac{1}{2} \cdot (-3) \cdot x^{-3-1} - 3 \cdot \frac{5}{4}x^{\frac{5}{4}-1} + 0 = 24x^5 - \frac{3}{2}x^{-4} - \frac{15}{4}x^{\frac{1}{4}}.$$

Example 2. Find a derivative of a function: $y = ctgx \cdot x^5$.

Solution.

$$y' = \left[ctgx \cdot x^5\right]' = \left| \begin{array}{l} (u \cdot v)' = u' \cdot v + u \cdot v' \\ u = ctgx, \quad v = x^5 \\ u' = (ctgx)' = -\dfrac{1}{\sin^2 x} \\ v' = \left(x^5\right)' = 5x^4 \end{array} \right| = (ctgx)' \cdot x^5 + ctgx \cdot \left(x^5\right)' =$$

$$= \left(-\frac{1}{\sin^2 x} \right) \cdot x^5 + ctgx \cdot 5x^4 = -\frac{x^5}{\sin^2 x} + ctgx \cdot 5x^4.$$

Example 3. Find a derivative of a function: $y = \dfrac{x^3}{arctgx}$.

Solution.

$$y' = \left(\frac{x^3}{arctgx} \right)' = \left| \begin{array}{l} \left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2} \\ u = x^3, v = arctgx \\ u' = \left(x^3\right)' = 3x^2 \\ v' = (arctgx)' = \dfrac{1}{1+x^2} \end{array} \right| = \frac{\left(x^3\right)' \cdot arctgx - x^3 \cdot (arctgx)'}{(arctgx)^2} =$$

$$= \frac{3x^2 arctgx - x^3 \cdot \dfrac{1}{x^2+1}}{arctg^2 x}.$$

Example 4. Find a derivative of a composite function:

$$y' = (\sin 7x)' = \cos 7x \cdot (7x)' = 7\cos 7x$$

$$y' = \left(2^{\cos x}\right)' = 2^{\cos x} \cdot \ln 2 \cdot (\cos x)' = 2^{\cos x} \cdot \ln 2 \cdot (-\sin x)$$

TASKS Find derivatives of functions:

1) x; 2) x^2; 3) x^3; 4) x^4; 5) x^5; 6) $\dfrac{x}{10}$; 7) $\dfrac{x^2}{3}$; 8) $5x^3$; 9) $\dfrac{7}{2}x^4$;

10) $2x^5 + 3x$; 11) $4x^2 - 7x^6 + 8$; 12) $\dfrac{8}{x^5}$; 13) $\sqrt[5]{x^2}$; 14) $\dfrac{5}{6}x \cdot \sqrt[5]{x^2}$, 15) $\dfrac{x^2}{x^7}$,

16) $x^6 \cdot x^3$, 17) $\left(x^5\right)^4$; 18) $y = x^2 \sin x$; 19) $y = \dfrac{x^2+2x}{3-4x}$. 20) $y = \dfrac{x^2+x-1}{x^2+1}$.

21) $y = \dfrac{x}{1-\cos x}$. 22) $y = 3\arcsin x - 4\sqrt{x}$. 23) $y = 5^x$; 24) $y = \sin x \cdot \arccos x$;

25) $y = \ln\left(x^2 - 3x\right)$; 26) $y = \cos^3 x$. 27) $y = \sin(1-2x)$ 28) $y = \sin\left(x^2\right)$

29) $y = \left(x^2 - 3x + 1\right)\left(x^2 + 2x - 1\right)$. **30)** $y = 10x^5$ **31) 2.** $y = \dfrac{1}{x^3}$.

32) $y = x^4 - \frac{4}{3}x^3 - 3x^2 + \frac{x}{3} + \sqrt{2}$. 33) $y = 3x^{-2} - \frac{5}{6}x^{-3} + 3$. 34) $y = x^{-5} - 9x^{-2} - 0{,}3x^{-1} + \frac{1}{2}$.

35) $y = \frac{5}{x} - \frac{4}{x^2} + \frac{5}{x^3} - \frac{6}{11x^4}$. **36)** $y = \frac{6x^5 - 7x^3 + x^2 - 5x + 3}{2x^4}$. **37)** $y = \sin x \cdot \arccos x$;

38) $y = 2x^6 - 5 \cdot 3^x + 4x - 7\log_2 x - \frac{5}{3}$ **39)** $y = \frac{x}{\sin x}$; **40)** $y = \frac{x^2}{\ln x}$; **41)** $y = \frac{3x+2}{2x+3}$;

42) $y = \left(1 - 5x + x^3\right) \cdot arctgx$; **43)** $y = 3\arcsin x - 4\sqrt{x}$. **44)** $y = 7^x$. **45)** $y = \ln x$.

46) $y = \sin x$. **48)** $y = xe^x$. **49)** $y = x^3 + 2x^2 - 4x - 3$ **50)** $y = \frac{x^2 - 2x + 3}{x + 2}$;

51) $y = x^4 - 8x^2 + 3$; **52)** $y = 2x^3 + 3x^2 - 12x + 1$