TOPIC: A DERIVATIVE

Find derivatives:

18. $y = x^4 - \frac{4}{3}x^3 - 3x^2 + \frac{x}{3} + \sqrt{2}.$ **19.** $y = 3x^{-2} - \frac{5}{6}x^{-3} + 3.$

20. $y = x^{-5} - 9x^{-2} - 0{,}3x^{-1} + \frac{1}{2}.$ **21.** $y = \frac{5}{x} - \frac{4}{x^2} + \frac{5}{x^3} - \frac{6}{11x^4}.$

22. $y = \frac{6x^5 - 7x^3 + x^2 - 5x + 3}{2x^4}.$ **23.** $y = x^{\frac{3}{2}} - 2x^{\frac{2}{3}} + 3x^{\frac{1}{3}}.$

24. $y = 3\sqrt{x} + 5x\sqrt[3]{x} - \sqrt[4]{x} + \frac{1}{x}.$ **25.** $y = 4\sqrt[4]{x} + 9x^2\sqrt[3]{x} - 3x\sqrt[5]{x^2} + \frac{7}{x^2}.$

26. $y = \left(x^2 - 3x + 1\right)\left(x^2 + 2x - 1\right).$ **27.** $y = (\sqrt{x} + 1)(\frac{1}{\sqrt{x}} - 1)$

28. $y = (6\sqrt[3]{x} - \frac{1}{x^2})(7x - 3),$ **29.** $y = (3x^2 - \frac{1}{x^2})(\sqrt[3]{x} + 0{,}1x)$ **30.** $y = \frac{x}{x^2 + 1}.,$

31. $y = \frac{x^2 + 2x}{3 - 4x}.,$ **32.** $y = \frac{1 + \sqrt{x}}{1 - \sqrt{x}}.,$ **33.** $y = \frac{x^2 + x - 1}{x^2 + 1}.,$

34. $y = \frac{x^2 + 1}{3(x^2 - 1)} + (x^2 - 1)(1 - x).,$ **35.** $y = \frac{\sqrt[3]{x} - 2}{\sqrt[3]{x} + 2}.,$ **36.** $y = \left(5x + 2\right)^4,$

37. $y = (\frac{1}{9}x^3 - \frac{4}{x} + 6)^6$ **38.** $y = \sqrt{1 - x^2}.,$ **39.** $y = (1 - 4\sqrt{x})^4.,$

40. $y = \frac{4}{(2x^2 + x - 1)^2}.,$ **41.** $y = \frac{1}{\sqrt[3]{1 + x^3}}.,$ **42.** $y = \frac{1}{\sqrt{x^4 - x^2 + 3}}.,$

43. $y = \frac{1}{\sqrt[3]{2x - 1}} + \frac{5}{\sqrt[4]{(x^2 + 2)^3}}.,$ **44.** $y = \sqrt[5]{(2x^2 - 4x^3)^4}.,$ **45.** $y = \frac{1 - \sqrt[3]{2x}}{1 + \sqrt[3]{2x}}.,$

46. $y = \frac{x}{1 - \cos x}.,$ **47.** $y = \frac{\sin x}{x} + \frac{x}{\sin x}.,$ **48.** $y = 3\sin x - 5x\cos x.,$

49. $y = 2\sqrt{x}\sin x - \frac{\cos x}{x}.,$ **50.** $y = \frac{-5\sin x}{2 - 3\cos x}.,$ **51.** $y = \frac{4\cos x}{tgx - 2x}.,$ **52.** $y = \frac{\sqrt[3]{x^2}}{ctgx - 1}.$

53. $y = \sqrt[5]{x^4}ctgx - \frac{tgx}{x}.,$ **54.** $y = \sin\left(1 - 2x\right),$ **55.** $y = \sin\left(x^2\right),$ **56.** $y = \sqrt[3]{\sin^2 x}.,$

57. $y = \cos^3 x.,$ **58.** $y = 3\sin^4 x - \sin^2 x.,$ **59.** $y = \frac{2}{\cos 5x}.,$ **60.** $y = \sin\frac{1}{x} + \cos\sqrt{x}.,$

61. $y = \sin\sqrt{1 + x^2} + \sqrt{\sin 2x}.,$ **62.** $y = \frac{\sqrt{x}}{\sin\left(2x + 1\right)}.,$ **63.** $y = \sqrt{\sin x + 2tgx}.,$

64. $y = ctg\sqrt[3]{x^2}.,$ **65.** $y = tg^2 x - ctg\left(x^2\right)$ **66.** $y = \frac{1 - \sin 2x}{1 + \sin 2x}.,$ **67.** $y = \left(1 + \sin^2 x\right)^3,$

68 $y = \sqrt{2 - \mathrm{tg}(x - \frac{1}{x})}$ **,69.** $y = \sin^5\left(\cos 5x\right),$ **70.** $y = 3\arcsin x - 4\sqrt{x}.,$

71 $y=\sin x\cdot\arccos x$, **72.** $y=\sqrt{2}\arccos x-\frac{2}{\arcsin x}.$, **73.** $y=\frac{2arctgx-x}{3arctgx}.$,

74. $y=(\sqrt[3]{x^2-1})\operatorname{arctg}x$, **75.** $y=\frac{9\sqrt[3]{x^2}+2}{\arccos x}.$, **76.** $y=\arcsin\frac{x}{2}.$, **77.** $y=arctgx^2.$,

78. $y=arctg\left(3-x^2\right)$, **79.** $y=2\sqrt[4]{\arcsin^3 x}.$, **80.** $y=\arccos\frac{1}{\sqrt{x}}.$,

81. $y=\sqrt{1-\arccos^2 x}.$, **82.** $y=arctg\,(x-\sqrt{1+x^2}).$, **83.** $y=\sqrt[3]{\arcsin\left(2x+1\right)}.$,

84. $y=10x\cdot\arccos\sqrt{1-5x}$ **85.** $y=\sqrt[4]{\arcsin\sqrt{x^2+2x}}.$, **86.** $y=\frac{3\ln x}{x^2}.$, **87.** $y=\frac{1-\ln x}{1+\ln x}.$

88. $y=\ln^3 x.$, **89.** $y=\sqrt[3]{2+\ln x}.$,**90.** $y=\ln\left(x^2-3x\right)$, **91.** $y=\ln\,(x+\sqrt{1+x^2})$.

Find derivatives of implicit function:

106. $y=5x-2y+15$ **107.** $y^2-x^2+2y=0.$ **108.** $y^3-x^3+x^2y^2=0.$

109. $\sqrt{x}+\sqrt{y}=2.$ **110.** $x\cos y=y\sin x.$ **111.** $arctg\left(x+y\right)=x.$

112. $e^x+e^y-e^{xy}=1.$ **113.** $y=\cos\left(x+y\right).$ **114.** $2y\ln y=x.$ **115.** $xe^y-\sqrt{3}=y.$

117. $x^3+y^3-3y+3=0.$

Find derivatives of parametric functions:

120. $x=\ln t, y=t^2.$ **121.** $x=t^3+3t^2,\ y=3t^2-7.$

122. $x=arctgt,\ y=\ln\left(1+t^2\right).$ **123.** $x=a\left(\sin t-t\cos t\right), y=a\left(\cos t+t\sin t\right).$

124. $x=a\cos 2t, y=a\sin^2 t.$ **125.** $x=\arccos\sqrt{t},\ y=\sqrt{t-t^2}.$

126. $x=\arcsin t,\ y=\ln\left(1-t^2\right).$ **127.** $x=\arcsin t,\ y=\sqrt{1-t^2}.$

Find derivatives of power exponential functions:

128. $y=\left(x+1\right)\left(2x+1\right)\left(3x+1\right).$ **129.** $y=\left(3x-4\right)^4\left(2x+7\right)^5\left(x-1\right)^6.$

130. $y=\frac{\left(x+2\right)^2}{\left(x+1\right)^3\left(x+3\right)^4}.$ **131.** $y=\frac{\left(x-2\right)^3\sqrt{5x+1}}{\left(x+1\right)^4}.$ **132.** $y=\frac{\left(x+1\right)^3\sqrt[4]{x-2}}{\sqrt[4]{\left(x-5\right)^3}}.$

133. $y=\frac{e^x\arcsin x}{x^2-1}.$ **134.** $y=\frac{\ln^3 x}{\sqrt{x-1}\sin 2x}.$ **135.** $y=x^{\sin 2x}.$ **136.** $y=x^{x+1}$

137. $y=\left(\sin x\right)^x$ **138.** $y=\left(x^2+1\right)^{\sin x}$ **139.** $y=\left(x+1\right)^{\frac{2}{x}}.$ **140.** $y=x^{\frac{1}{x}}.$

9.143. Form equations of a tangent line and a normal line for a function $x^3+y^3-xy-7=0$ at the point $\left(1;2\right)$.

145 Form equations of a tangent line and a normal line for the function $y=x^3+2x^2-4x-3$ at the point $\left(-2;5\right)$.

146 Form equations of a tangent line and a normal line for the function $y=\frac{3-x}{2x-3}$ at the point with the abscissa $x_0=2$.

ANSWERS: 18. $4x^3-4x^2-6x+\frac{1}{3}..$ **19.** $-\frac{6}{x^3}+\frac{5}{2x^4}..$ **20.** $-\frac{5}{x^6}+\frac{18}{x^3}+\frac{0,3}{x^2}..$

21. $-\frac{5}{x^2}+\frac{8}{x^3}-\frac{15}{x^4}+\frac{24}{11x^5}..$ **22.** $3+\frac{7}{2x^2}-\frac{1}{x^3}+\frac{15}{2x^4}-\frac{6}{x^5}..$ **23.** $\frac{3}{2}\sqrt{x}-\frac{4}{3\sqrt[3]{x}}+\frac{1}{\sqrt[3]{x^2}}..$

24. $\frac{3}{2\sqrt{x}}+\frac{20}{3}\sqrt[3]{x}-\frac{1}{4\sqrt[4]{x^3}}-\frac{1}{x^2}.$ **25.** $\frac{1}{\sqrt[4]{x^3}}+21x\sqrt[3]{x}-\frac{21}{5}\sqrt[5]{x^2}-\frac{14}{x^3}.$

26. $4x^3-3x^2-12x+5..$ **27.** $-\frac{x+1}{x\sqrt{x}}..$ **28.** $56\sqrt[3]{x}-\frac{6}{\sqrt[3]{x^2}}+\frac{7}{x^2}-\frac{6}{x^3}..$

29. $7x\sqrt[3]{x}+\frac{1}{10x^2}+\frac{5}{3x^2\sqrt[3]{x^2}}+0{,}9x^2..$ **30.** $\frac{1-x^2}{(x^2+1)^2}..$ **31.** $\frac{6x-4x^2+6}{(3-4x)^2}..$

32. $\frac{1}{\sqrt{x}(1-\sqrt{x})^2}..$ **33.** $\frac{1+2x+3x^2-2x^3-x^4}{(1+x^2)^2}..$ **34.** $\frac{-4x}{3(x^2-1)^2}+1+2x-3x^2..$

35. $\frac{4}{3\sqrt[3]{x^2}(\sqrt[3]{x}+2)^2}..$ **36.** $20(5x+2)^3..$ **37.** $6(\frac{x^3}{9}-\frac{4}{x}+6)^5(\frac{x^2}{3}+\frac{4}{x^2}).$

38. $\frac{-x}{\sqrt{1-x^2}}..$ **39.** $-8(1-4\sqrt{x})^3\cdot\frac{1}{\sqrt{x}}.$ **40.** $\frac{-8(4x+1)}{(2x^2+x-1)^3}..$ **41.** $-\frac{x^2}{\sqrt[3]{(1+x^3)^4}}..$

42. $\frac{x-2x^3}{\sqrt{(x^4-x^2+3)^3}}..$ **43.** $\frac{-2}{3\sqrt[3]{(2x-1)^4}}-\frac{15x}{2\sqrt[4]{(x^2+3)^7}}..$ **44.** $\frac{16(x-3x^2)}{5\sqrt[5]{2x^2-4x^3}}..$

45. $\frac{-4}{3\sqrt[3]{4x^2}(1+\sqrt[3]{2x})^2}..$ **46.** $\frac{1-\cos x-x\sin x}{(1-\cos x)^2}..$ **47.** $\frac{x(x\cos x-\sin x)(\sin^2 x-x^2)}{x^2\sin^2 x}..$

48. $5x\sin x-2\cos x..$ **49.** $\frac{\sin x+2x\cos x}{\sqrt{x}}+\frac{x\sin x+\cos x}{x^2}..$ **50.** $\frac{5(3-2\cos x)}{(2-3\cos x)^2}..$

51. $\frac{4(x\sin 2x-\sin^2 x+\cos 2x)}{\cos x(tgx-2x)^2}..$ **52.** $\frac{2\sin^2 x(ctgx-1)+3x}{\sqrt[3]{x}\sin^2 x(ctgx-1)^2}..$

53. $\frac{2\sin 2x-5x}{5\sqrt[5]{x}\sin^2 x}-\frac{x-\sin x\cos x}{x^2\cos^2 x}..$ **54.** $-2\cos(1-2x)..$ **55.** $2x\cos x^2..$

56. $\frac{2\cos x}{3\sqrt[3]{\sin x}}..$ **57.** $-3\cos^2 x\sin x..$ **58.** $12\sin^3 x\cos x-\sin 2x..$ **59.** $\frac{10\sin 5x}{\cos^2 5x}..$

60. $-\frac{1}{x^2}\cos\frac{1}{x}-\frac{\sin\sqrt{x}}{2\sqrt{x}}..$ **61.** $\frac{x}{\sqrt{1+x^2}}\cos\sqrt{1+x^2}+\frac{\cos 2x}{\sqrt{\sin 2x}}..$

62. $\frac{\sin(2x+1)-4x\cos(2x+1)}{2\sqrt{x}\sin^2(2x+1)}..$ **63.** $\frac{\cos^3 x+2}{2\cos^2 x\sqrt{\sin x+2tgx}}..$ **64.** $\frac{-2}{3\sqrt[3]{x}}\cdot\frac{1}{\sin^2\sqrt[3]{x^2}}.$

65. $\frac{2tgx}{\cos^2 x}+\frac{2x}{\sin^2 x^2}..$ **66.** $\frac{-4\cos 2x}{(1+\sin 2x)^2}..$ **67.** $3(1+\sin^2 x)^2\sin 2x..$

68. $-\frac{x^2+1}{2x^2\cos^2(x-\frac{1}{x})\sqrt{2-\mathrm{tg}(x-\frac{1}{x})}}.$ **69.** $-25\sin^4 x(\cos 5x)\cdot\cos(\cos 5x)\cdot\sin 5x.$

70. $\frac{3}{\sqrt{1-x^2}}-\frac{2}{\sqrt{x}}$.. **71.** $\cos x\arccos x-\frac{\sin x}{\sqrt{1-x^2}}$.. **72.** $\frac{-\sqrt{2}\arcsin^2 x+2}{\sqrt{1-x^2}\,arc\sin^2 x}$.. **73.**

$\frac{x-(1+x^2)\,arctgx}{3(1+x^2)\,arctg^2 x}$.. **74.** $\frac{2(1+x^2)\,arctgx+3x}{3\sqrt[3]{x}(1+x^2)}$.. **75.** $\frac{6\sqrt{1-x^2}\arccos x+9x+2\sqrt[3]{x}}{\sqrt[3]{x}\sqrt{1-x^2}\arccos^2 x}$..

76. $\frac{1}{\sqrt{4-x^2}}$.. **77.** $\frac{2x}{1+4x^2}$.. **78.** $\frac{-2x}{1+(3-x^2)^2}$.. **79.** $\frac{3}{2}\frac{1}{\sqrt[4]{\arcsin x}}\cdot\frac{1}{\sqrt{1-x^2}}$.

80. $\frac{1}{2x\sqrt{x-1}}$.. **81.** $\frac{\arccos x}{\sqrt{1-x^2}\cdot\sqrt{1-\arccos^2 x}}$. **82.** $\frac{1}{2(x^2+1)}$.. **83.** $\frac{2}{3}\cdot\frac{1}{\sqrt[3]{\arcsin^2(2x+1)}}\cdot\frac{1}{\sqrt{-4x^2-4x}}$.

84. $10\arccos\sqrt{1-5x}+\frac{25x}{\sqrt{5x}\sqrt{1-5x}}$..

85. $\frac{1}{4}\cdot\frac{1}{\sqrt[4]{\arcsin^3\sqrt{x^2+2x}}}\cdot\frac{1}{\sqrt{1-x^2-2x}}\cdot\frac{x+1}{\sqrt{x^2+2x}}$. **86.** $\frac{3-6\ln x}{x^3}$.. **87.** $\frac{-2}{x\left(1+\ln x\right)^2}$..

88. $\frac{3\ln^2 x}{x}$.. **89.** $-\frac{1}{3}\cdot\frac{1}{x\sqrt[3]{(2+\ln x)^2}}$. **90.** $\frac{2x-3}{x^2-3x}$.. **91.** $\frac{1}{\sqrt{1+x^2}}$.. **92.** $y''=12x^2-8$. **106.** $y'=\frac{5}{3}$.

107. $y'=\frac{x}{y+1}$. **108.** $y'=\frac{3x^2-2xy^2}{2x^2y+3y^2}$. **108.** $y'=-\frac{\sqrt{y}}{\sqrt{x}}$. **110.** $y'=\frac{\cos y-y\cos x}{\sin x+x\sin y}$. **111.** $y'=(x+y)^2$.

112. $y'=\frac{e^x-ye^{xy}}{e^y-xe^{xy}}$. **113.** $y'=-\frac{\sin(x+y)}{1+\sin(x+y)}$. **114.** $y'=\frac{1}{2(1+\ln y)}$. **115.** $y'=\frac{e^y}{2-y}$.

116. $y''=\frac{y^2+x^2(y')^2}{x^2y}$. **117.** $y''=\frac{2(x+y\cdot(y')^2)}{1-y^2}$. **120.** $y''=4t^2$. **121.** $y''=\frac{-4}{(t+2)^2}$.

122. $y''=2(t^2+1)$. **123.** $y''=-\frac{1}{at\sin^3 t}$. **124.** $y''=0$. **125.** $y''=-4\sqrt{1-t^2}$. **126.** $y''=\frac{2}{t^2-1}$.

127. $y''=-\sqrt{1-t^2}$. **128.** $y'=y(\frac{1}{x+1}+\frac{2}{2x+1}+\frac{3}{3x+1})$. **129.** $y'=y(\frac{12}{3x-4}+\frac{10}{2x+7}-\frac{6}{x-1})$.

130. $y'=y(\frac{2}{x+2}-\frac{3}{x+1}-\frac{4}{x+3})$. **131.** $y'=y(\frac{3}{x-2}+\frac{5}{2(5x+1)}-\frac{4}{x+3})$.

132. $y'=y(\frac{3}{x+1}+\frac{1}{4(x-2)}-\frac{3}{4(x-5)})$. **133.** $y'=y(1+\frac{1}{\arcsin x\cdot\sqrt{1-x^2}}-\frac{2x}{x^2-1})$.

134. $y'=y(\frac{3}{x\ln x}-\frac{1}{2(x-1)}-2c\operatorname{tg}2x)$. **135.** $y'=y(2\cos 2x\cdot\ln x+\frac{\sin 2x}{x})$

136. $y'=y(\frac{x\ln x+x+1}{x})$. **137.** $y'=y(\ln\sin x+xc\operatorname{tg}x)$. **138.** $y'=y(\cos x\cdot\ln(x^2+1)+\frac{2x\sin x}{x^2+1})$.

139. $y'=2y\frac{x-(x+1)\ln(x+1)}{x^2(x+1)}$. **140.** $y'=y(\frac{1-\ln x}{x^2})$. **141..** **142.** **143.** $\left(1;-3\right)$.

145. $y-5=0,\ x+2=0.$ **146.** $y+3x-7=0.$