Tasks

Task 1. For the pairs of matrices below say whether it is possible to add (subtract) them together and then, where it is possible, derive the matrices $C = A + B$, $D = A - B$, $F = 3A - 4B$, $G = (-3)A + \frac{1}{2}B$:

1) $A = \begin{pmatrix} 0 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$;

2) $A = \begin{pmatrix} 1 & 4 & 2 & 3 \\ -3 & 2 & -5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 0 & 2 & -5 \\ 6 & 1 & 3 & 1 \end{pmatrix}$;

3) $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & -4 & -3 \\ -1 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 6 & -2 \\ -1 & 5 & 3 \end{pmatrix}$.

Task 2. In each of the following cases, determine whether the products *AB* and *BA* are both defined; if so, determine also whether *AB* and *BA* have the same number of rows and the same number of columns; if so, determine also whether *AB* = *BA:*

a) $A = \begin{pmatrix} 0 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$;

b) $A = \begin{pmatrix} 1 & -1 & 5 \\ 3 & 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 6 \\ 1 & 5 \end{pmatrix}$;

c) $A = \begin{pmatrix} 3 & 1 & -4 \\ -2 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

Task 3. Calculate A^2 and A^3, where

1) $A = \begin{pmatrix} 2 & -5 \\ 3 & 1 \end{pmatrix}$; 2) $A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 3 & 2 \\ 0 & 2 & -7 \end{pmatrix}$.

Task 4. Carry out the operations $2(A+B)B^T$, $3B(B-2A)$, $(3A-2B)A^T$ on the given matrices and check the following properties: a) $(A+B)C = AC+BC$, b) $C(A+B) = CA+CB$, c) $A(BC) = (AB)C$.

1) $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & -4 & -3 \\ -1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -3 & 0 \\ 3 & 2 & -1 \\ -2 & -1 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 4 \\ -2 & 0 & 3 \\ 5 & 6 & 4 \end{pmatrix}$;

2) $A = \begin{pmatrix} 4 & -1 & 5 \\ 6 & 4 & -3 \\ -2 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 3 \\ 5 & 3 & -5 \\ -4 & -1 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 3 & -1 & 3 \\ 5 & 3 & -5 \\ -4 & -1 & 4 \end{pmatrix}$.

Task 5. Consider the six matrices ,

$A = \begin{pmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 1 & 3 \\ 1 & 1 & 5 \\ 3 & 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 7 \\ 2 & 1 & 2 \\ 1 & 3 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $F = \begin{pmatrix} 1 & 7 & 2 & 9 \\ 9 & 2 & 7 & 1 \end{pmatrix}$;

$G = \begin{pmatrix} -4 & 7 & 2 \end{pmatrix}$

a) Calculate all possible products.
b) Transpose these matrices.

Task 6. Find the matrix $C = AB - BA$, if

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 3 & 1 & -2 \\ 3 & -2 & 4 \\ -3 & 5 & -1 \end{pmatrix}.$$

Task 7. Find the matrix $C = A^2 - (A+B)(A-3B)$, if

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}, \; B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Task 8. Find the inverse matrix for the given matrix and check $A \cdot A^{-1}$ or $A^{-1} \cdot A$:

$$1)\ A=\begin{pmatrix} 2 & 8 & -3 \\ -1 & -7 & 4 \\ -3 & -6 & 2 \end{pmatrix};\quad 2)\ A=\begin{pmatrix} 3 & 2 & -3 \\ 5 & 4 & 1 \\ -6 & 3 & 1 \end{pmatrix};\quad 3)\ A=\begin{pmatrix} 2 & 2 & 1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix};$$

$$4)\ A=\begin{pmatrix} -6 & 9 & 0 \\ 1 & 2 & 3 \\ 11 & 5 & 7 \end{pmatrix};\quad 5)\ A=\begin{pmatrix} 3 & -1 & 1 \\ 1 & 0 & 2 \\ 2 & 2 & 1 \end{pmatrix};\quad 6)\ A=\begin{pmatrix} 6 & 3 & 2 \\ 7 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix}.$$

Task 9. Calculate the determinant of the given matrices on the base of Sarrus formula and check the result using the theorem concerning the decomposition of the determinant in row 1 (2 or 3) and column 1 (2 or 3):

$$1)\ A=\begin{pmatrix} 2 & 3 & 3 \\ 1 & 0 & -2 \\ -1 & -4 & 1 \end{pmatrix};\ 2)\ A=\begin{pmatrix} 1 & 2 & 1 \\ 0 & -2 & -3 \\ -2 & -3 & 2 \end{pmatrix};\quad 3)\ A=\begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & -2 \\ -1 & -1 & 4 \end{pmatrix};$$

$$4)\ B=\begin{pmatrix} 2 & 3 & 3 \\ 3 & 2 & -4 \\ -4 & 0 & 2 \end{pmatrix};\ 5)\ B=\begin{pmatrix} 5 & 6 & 1 \\ 4 & 0 & -1 \\ -3 & -1 & 4 \end{pmatrix};\quad 6)\ B=\begin{pmatrix} 5 & 1 & 2 \\ 7 & 2 & -3 \\ -1 & 0 & 6 \end{pmatrix}.$$

Task 10. Calculate determinants:

1) using Laplace's theorem and property 5;

2) reducing them to the triangular form and using property 7.

(a) $\begin{vmatrix} 2 & 4 & 1 \\ 3 & 6 & 2 \\ 4 & -1 & -3 \end{vmatrix}.$ **(b)** $\begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{vmatrix}.$

(c) $\begin{vmatrix} 1 & 2 & 2 & 2 \\ -2 & 2 & 2 & 2 \\ -2 & -2 & 3 & 2 \\ -2 & -2 & -2 & 4 \end{vmatrix}.$ **(d)** $\begin{vmatrix} 1 & 2 & 2 & 3 \\ 3 & -1 & -4 & -6 \\ 1 & 0 & 0 & 1 \\ 1 & 2 & -1 & 7 \end{vmatrix}.$

Task 11. Solve equations:

(a) $\begin{vmatrix} 1 & 2 & 3 \\ 3 & x+5 & 4 \\ 4 & 3 & 2 \end{vmatrix} = 0.$ **(b)** $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0.$

Task 12. Calculate the following determinants:

1) $\begin{vmatrix} -1 & -2 & 3 & 2 \\ 2 & 3 & 4 & -2 \\ -1 & 1 & 0 & 1 \\ 0 & -1 & 3 & 1 \end{vmatrix}$; 2) $\begin{vmatrix} 2 & 3 & 1 & -2 \\ 1 & -2 & -2 & 1 \\ -2 & 1 & 0 & -1 \\ 3 & 0 & 3 & 1 \end{vmatrix}$; 3) $\begin{vmatrix} 2 & 1 & 2 & 1 \\ 2 & 1 & 3 & -2 \\ 0 & -1 & 2 & 2 \\ -1 & 0 & 4 & -3 \end{vmatrix}$;

4) $\begin{vmatrix} 2 & 0 & 1 & 1 \\ 1 & -1 & 0 & 2 \\ 3 & 1 & -1 & -2 \\ 4 & -2 & 3 & 1 \end{vmatrix}$; 5) $\begin{vmatrix} -1 & 0 & 1 & 2 \\ 2 & 1 & -1 & -4 \\ 3 & 2 & 0 & 1 \\ 4 & -2 & 1 & 2 \end{vmatrix}$; 6) $\begin{vmatrix} 2 & -1 & 1 & -3 \\ 1 & -2 & -3 & 1 \\ -2 & 2 & 0 & -1 \\ 3 & 0 & 4 & 1 \end{vmatrix}$.

Task 13. Find ranks of matrices with the help of elementary row operations:

(a) $A = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 2 & -1 & 3 & 5 \\ 1 & 10 & -6 & 1 \end{pmatrix}.$ **(b)** $A = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & -1 & 2 & 5 \\ 3 & -6 & 5 & 6 \end{pmatrix}.$

(c) $A = \begin{pmatrix} 2 & 1 & 11 & 2 \\ 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 2 & -1 & 5 & -6 \end{pmatrix}.$ **(d)** $A = \begin{pmatrix} 1 & 3 & 1 & 4 & 1 \\ 11 & 3 & 5 & 5 & 2 \\ 5 & 5 & 3 & 17 & 12 \\ 4 & 2 & 2 & 3 & 1 \end{pmatrix}.$